

# THE ASTRONOMICAL JOURNAL

FOUNDED BY B. A. GOULD

PUBLISHED BY THE AMERICAN ASTRONOMICAL SOCIETY

---

VOLUME 60

1955 October ~ No. 1232

NUMBER 9

---

## COLOR-MAGNITUDE DIAGRAMS FOR SEVEN GLOBULAR CLUSTERS

By HALTON C. ARP

*Abstract.* Magnitudes and colors of about 2000 stars in seven globular clusters have been measured. The resulting color-magnitude diagrams show that the form and population of the blue or horizontal sequence differ most from cluster to cluster. The number of RR Lyrae cepheids in a given cluster is correlated with the population gradient across the RR Lyrae gap in the horizontal sequence. Some evidence is presented that the place of the giant sequence in the color-magnitude diagram is correlated with the spectral characteristics of the giants and the characteristics of the variable stars present in the cluster.

Estimates of field stars to be expected indicate that at least some of the stars in the diagrams which fall off the major sequences are cluster members. Finally, the assumption that the RR Lyrae cepheids in all the clusters have the same absolute magnitudes and normal colors is reviewed.

### 1.0 *Introduction.*

The first magnitudes and colors in globular clusters (Shapley 1917, Hachenberg 1939, Greenstein 1939, Cuffey 1943) showed the bright stars to be different from stars previously encountered in the neighborhood of the sun, in galactic clusters, and arms of spiral nebulae. After Baade (1944) used them in his demonstration of the two kinds of populations, these same globular-cluster stars became the working definition of population type II. More extensive observations of globular clusters were therefore desirable in order to study the range in characteristics of this population type.

New photometric techniques have improved the accuracy of magnitude scales and particularly the difference quantity of color index. On the assumption that RR Lyrae variables in all clusters have the same intrinsic luminosity and color, absolute magnitudes and normal color indices of stars in different clusters can be intercompared very accurately. Since RR Lyrae variables appear to occur exclusively in a well-defined gap in the horizontal sequence of the color-magnitude diagram, it becomes possible to compare diagrams by simply superposing them at this gap.

With M<sub>3</sub> and M<sub>92</sub> (Arp, Baum, and Sandage, 1952 and 1953), such a procedure showed a small but significant displacement in color index between the respective giant branches. The present work was designed to study for a significant number of clusters not only the exact places of type II sequences in the color-magnitude diagram, but also for each sequence its density of stars, discreteness and possible fine-structure, as well as the relation to these sequences of variable stars and color-spectrum anomalies. The material presented here for M<sub>5</sub>, M<sub>13</sub>, M<sub>10</sub>, M<sub>15</sub>, and M<sub>2</sub>, in conjunction with the published work on M<sub>3</sub> and M<sub>92</sub>, brings to a total of seven the number of globular-cluster color-magnitude diagrams having a comparable sample of stars on a homogeneous photometric system.

The results mark some progress on several fundamental problems. First, the absolute magnitudes of the brighter stars in the type II globular clusters have been calibrated relative to the RR Lyrae cepheids for a statistically reliable sample of stars. The globular-cluster giants, of course, are the brightest non-variable distance criterion in pure type II systems. Secondly, data on evolution and initial conditions begin to accumulate in the physical differences and similarities between clusters. Finally, such a sample of

globular clusters furnishes a reliable standard for comparison with other possible kinds of type II population such as the nucleus of the galaxy and certain nearby external systems.

## 2.0 Observational Procedure.

Photographic measures were made on IIa-O plates behind a GG<sub>1</sub> filter, and photovisual on 103a-D plates behind a GG<sub>11</sub> filter. Photoelectric calibration in each cluster ensured systematic accuracies of a few hundredths of a magnitude. These photoelectric sequences, color equations,

and measuring techniques are the same as described in a previous paper on type II cepheids (Arp 1955).

Except for M<sub>10</sub>, plates were taken at the Newtonian focus of the 100-inch telescope. The aperture was diaphragmed to 58 inches which gave a field of 7' radius free from comatic errors of more than 0.01 mag. M<sub>10</sub> was measured on plates taken at the 60-inch which was diaphragmed to 32 inches, giving a usable field of 11.5' radius.

The above procedure gives individual iris-

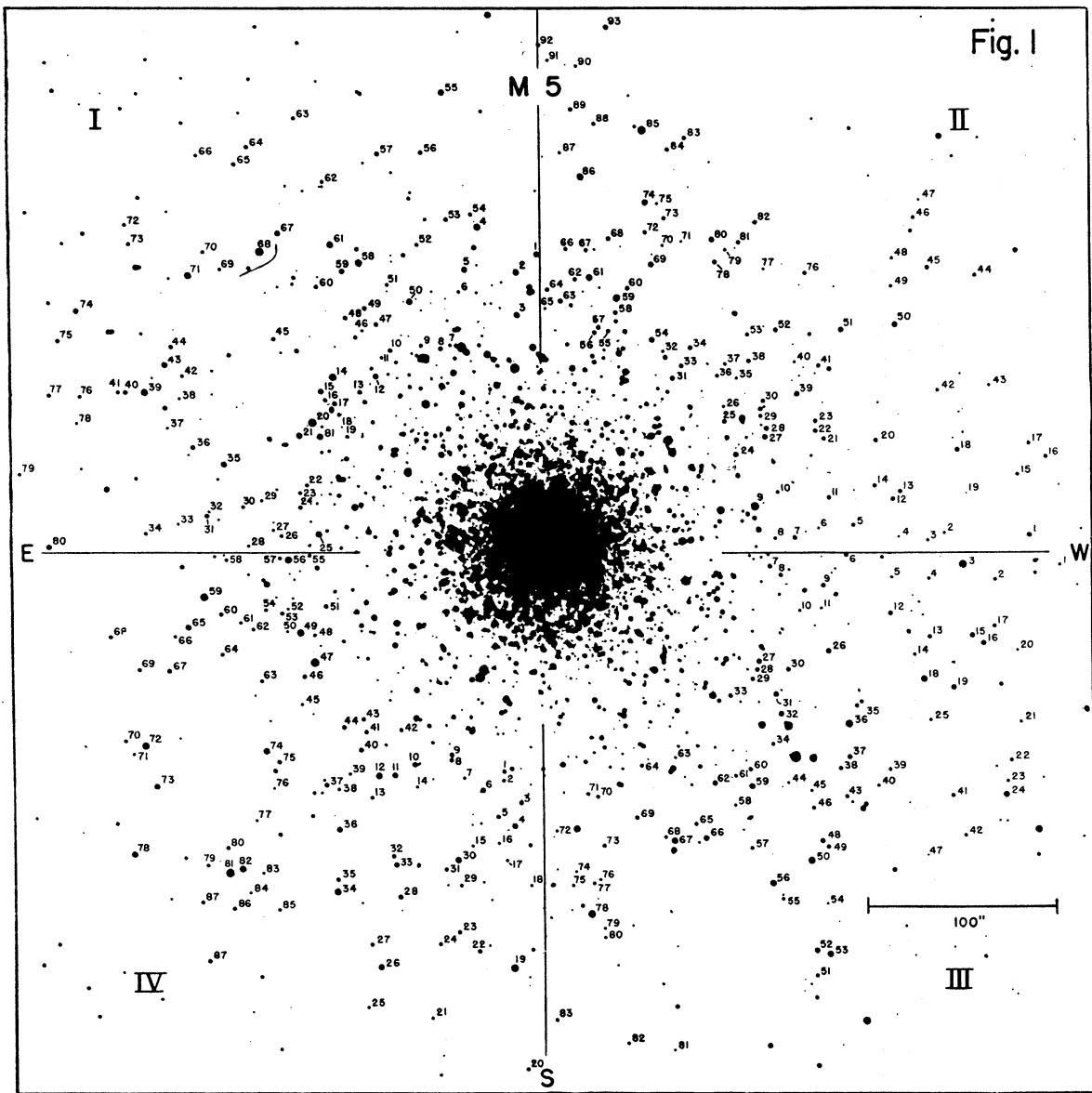


Figure 1. Identification chart for M<sub>5</sub>. Numbers refer to Table I.

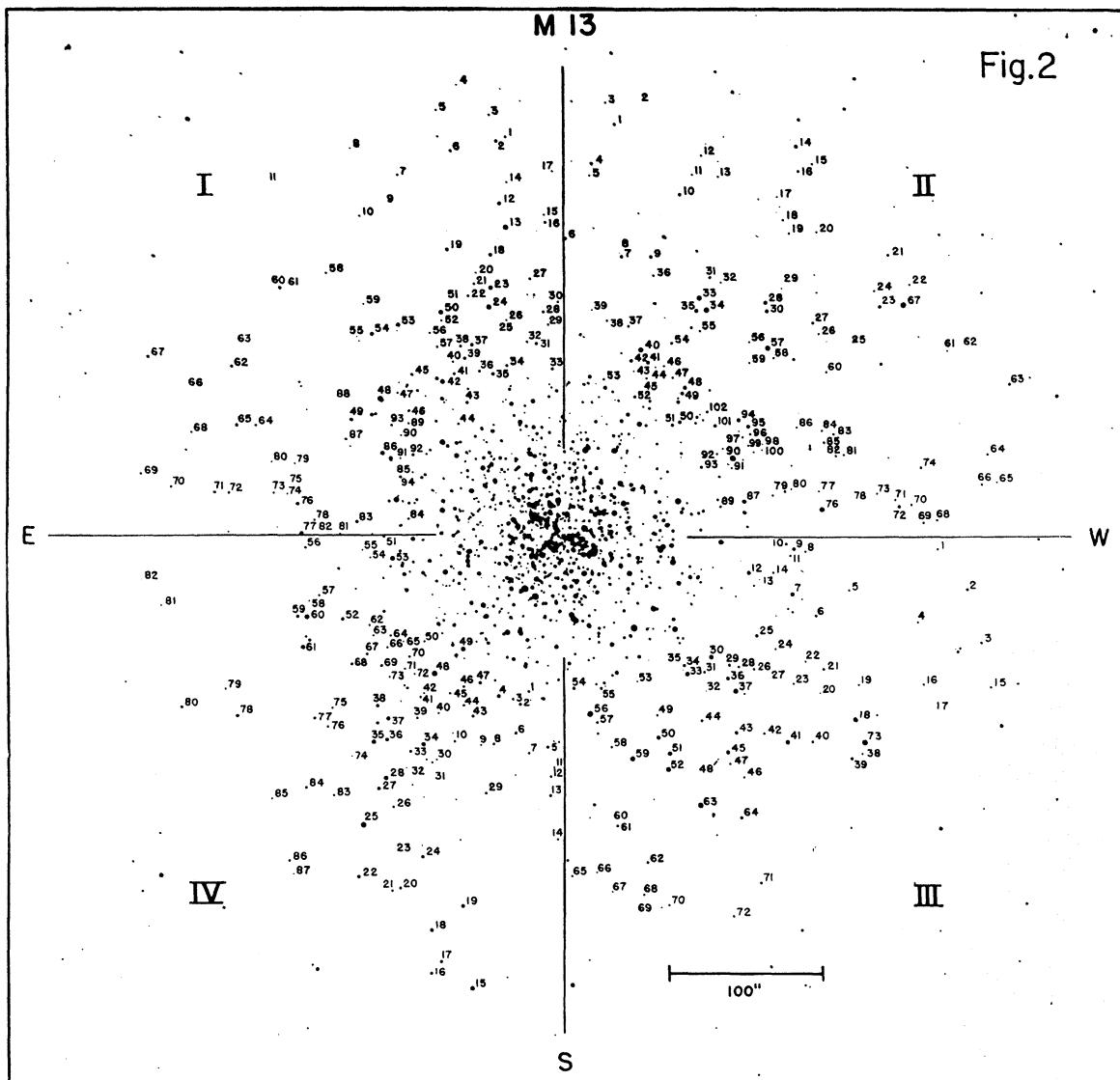


Figure 2. Identification chart for M13. Numbers refer to Table I.

photometer measures under the best conditions having probable errors of 0.02 for  $m_{pg}$ , 0.03 for  $m_{pv}$ , on plates taken at the 60-inch. Plates taken with the 100-inch give measures of somewhat greater accuracy, around 0.015 and 0.02, respectively.

**2.1 Selection of stars.** In each cluster an annulus centered on the cluster was chosen in which to measure the stars. The inner limit was placed as close to the center as possible without incurring measurable effects of background light. A limiting photovisual magnitude about one magnitude

fainter than the RR Lyrae stars was estimated and the outer circle was then drawn so that the total number of stars in the annulus, to this limiting magnitude, was about 300. All single, uncrowded stars were then measured. This procedure furnished diagrams which were as free as possible from non-cluster stars.

A few stars fainter than the limit were inevitably included and sometimes additional faint blue stars were purposely picked to outline the horizontal sequence to fainter limits. Consequently the number of stars varies from diagram

to diagram. Table III, however, gives the exact number of stars in each diagram brighter than  $M_{pv} = +0.7$  mag. (0.6 mag. below the gap in the horizontal sequence), enabling percentage populations of any features to be compared between clusters. With the exception of M92 the diagrams contain samples sufficiently comparable to allow visual intercomparison.

**2.2 Zero point and reddening.** The absolute magnitude of the RR Lyrae stars at mean light was assumed to be  $M_{pg} = 0.0$ ,  $M_{pv} = -0.1$  mag. As described in a previous paper (Arp 1955) the

center of the gap in the horizontal sequence is considered equivalent to this point, and one or both criteria were used to establish the zero point in each cluster. The adopted moduli are summarized in Table V and discussed for individual clusters in Section 3.

### 3.0 Color-Magnitude Diagrams.

Final magnitudes and colors of all stars measured in five clusters are catalogued in Table I. An identification chart is provided for each cluster (Figures 1-5). The catalogue of M3 stars has

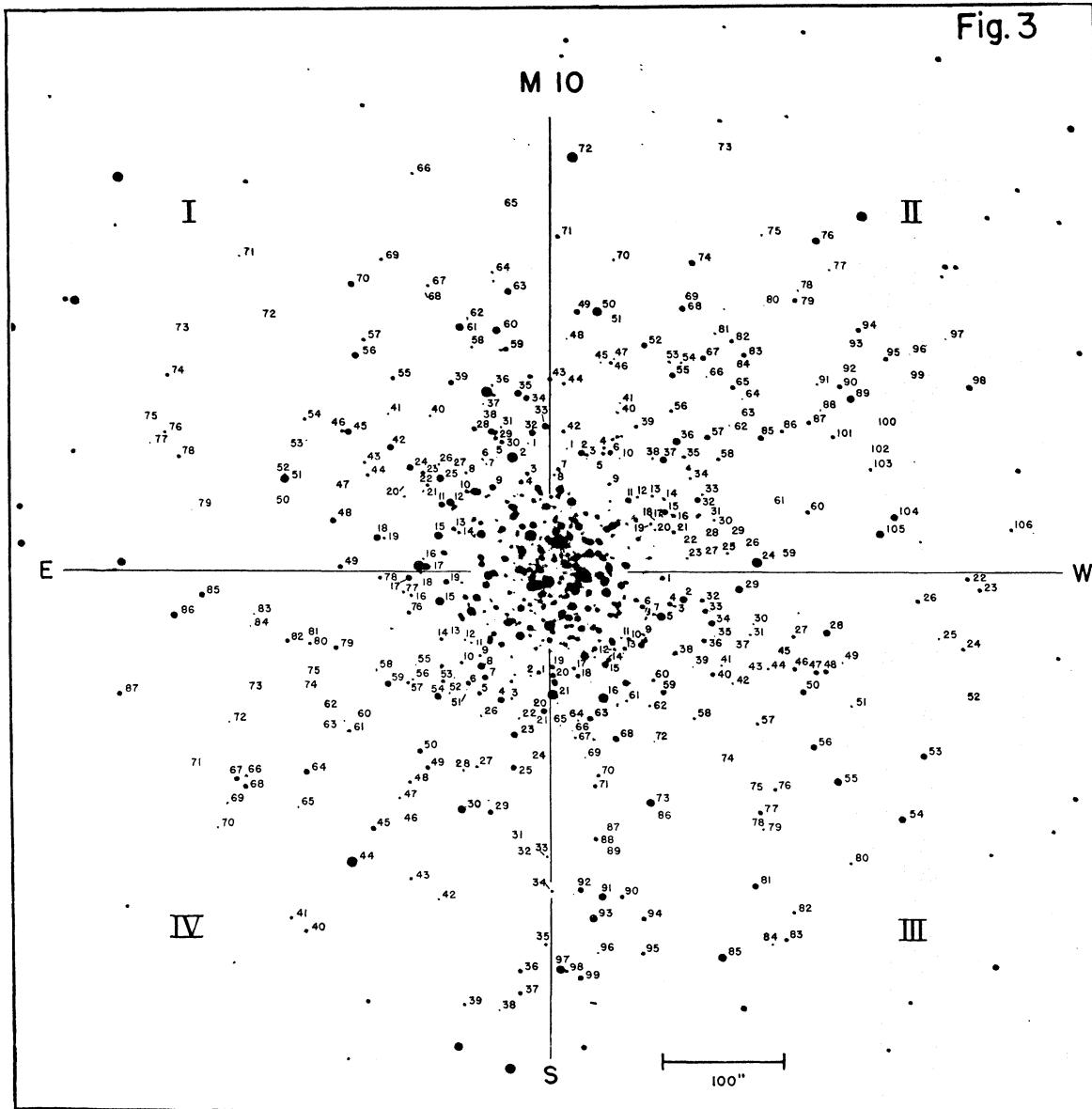
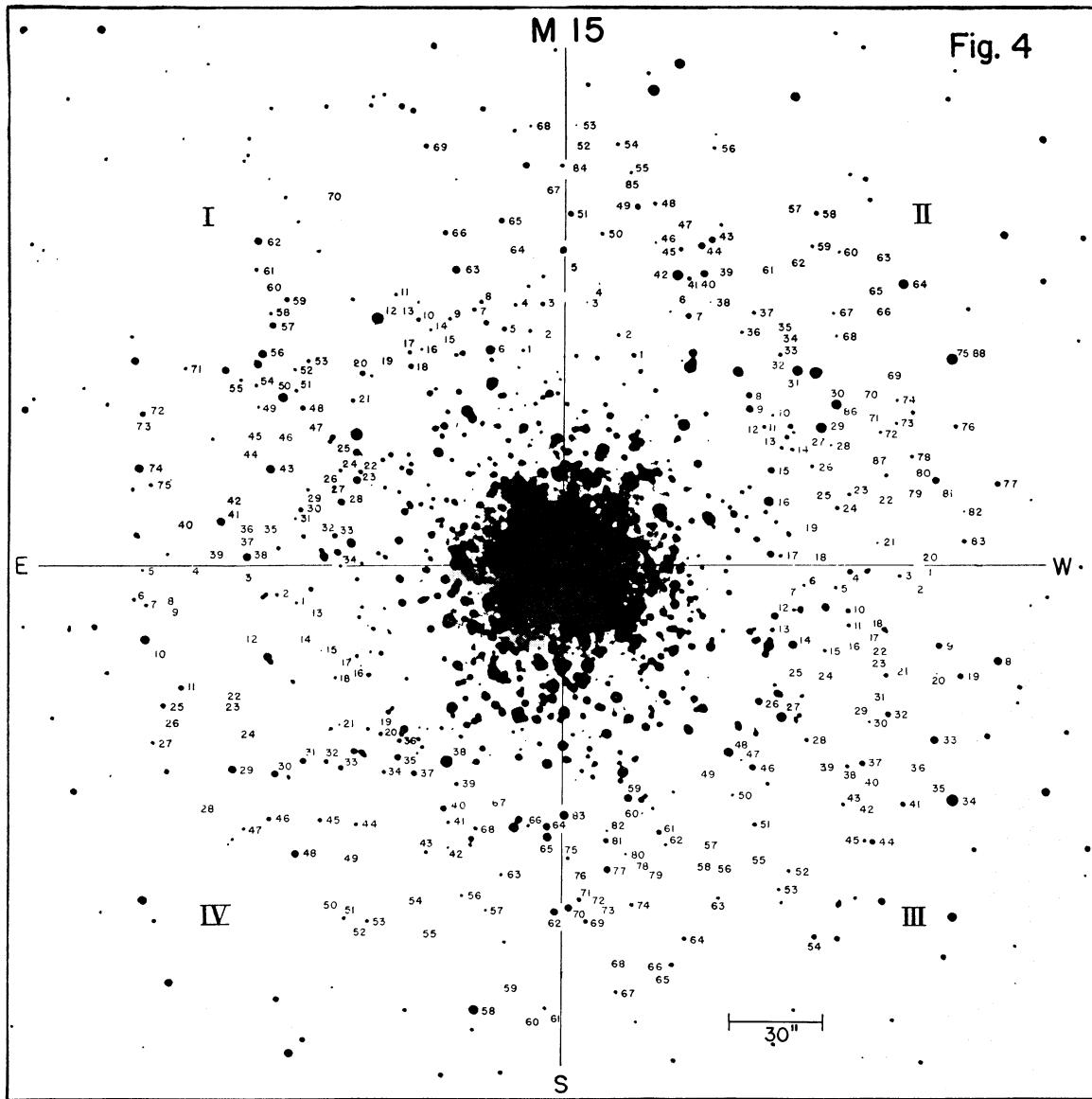


Figure 3. Identification chart for M10. Numbers refer to Table I.



variables known in any cluster and shows the greatest population on the red side of the gap of any of the clusters here. The horizontal sequence is well defined and the blue end effectively stops 0.7 mag. below the gap.

The zero point of luminosity in this cluster was judged solely from the gap. Recent measures by Roberts and Sandage (1955) show this to coincide accurately with the RR Lyrae stars at mean light.

**3.2 M<sub>5</sub>.** (Figure 7.) The general aspect of the M<sub>5</sub> diagram is very similar to that of M<sub>3</sub>. The displacement of the blue end of the gap from

C.I. = 0.0 mag. indicates +0.16 mag. reddening, however, and makes the terminus of the giant branch occur at a normal color index of +1.4 mag. Also the giant branch has appreciable slope at the magnitude of the gap.

Next to M<sub>3</sub> and  $\omega$  Centauri, M<sub>5</sub> has the largest number of RR Lyrae cepheids known in a cluster. The extension of the horizontal sequence to the red end of the gap, as in M<sub>3</sub>, is well marked. The blue end appears to stop about 0.9 mag. below the gap.

The 25- and 26-day cepheids were too bright to be included in the diagram. The cross refers

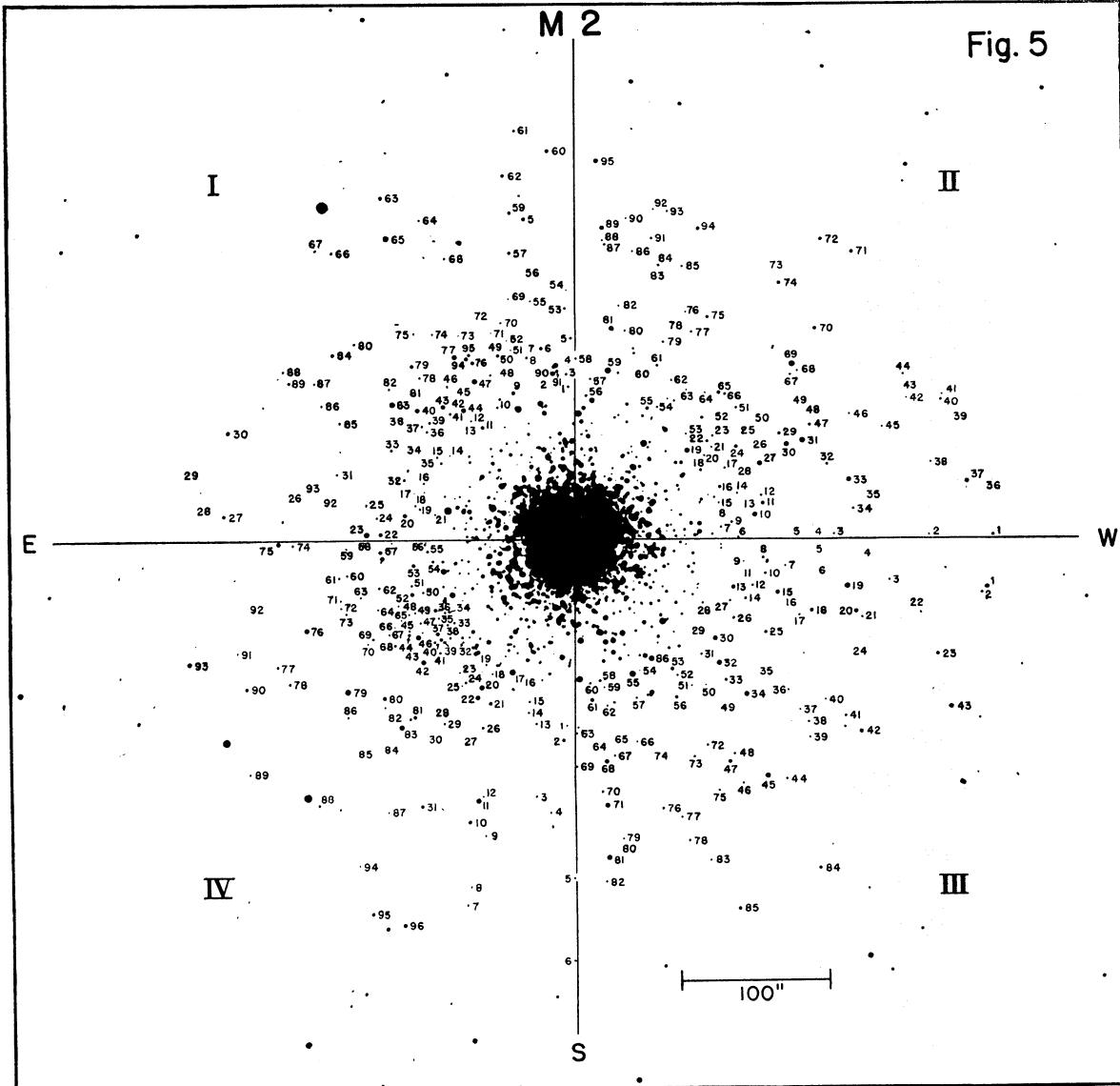


Figure 5. Identification chart for M<sub>2</sub>. Numbers refer to Table I.

TABLE I. COLOR-MAGNITUDE CATALOGUE FOR GLOBULAR CLUSTERS  
(One measure in each color for all stars unless noted.)

M <sup>2</sup>															
No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.	No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.	No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.	No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.
Sector I															
1	16.56	15.85	+ .71	74	16.19	16.18	+ .01	51	16.57	15.92	+ .65	28	15.97	16.03	- .06
2	17.37	16.68	+ .69	75	16.07	16.09	- .02	52	17.54	17.01	+ .53	29	17.55	16.89	+ .66
3	16.16	15.81	+ .35	76	15.76	14.96	+ .80	53	16.05	16.12	- .07	30	15.05	14.30	+ .75
4	17.20	16.54	+ .66	77	15.09	14.13	+ .96	54	16.61	15.93	+ .68	31	16.03	16.08	- .05
5	16.45	15.69	+ .76	78	16.73	16.15	+ .58	55	16.07	15.24	+ .83	32	14.97	14.01	+ .96
6	15.85	15.14	+ .71	79	16.10	15.10	+ 1.00	56	16.46	15.66	+ .80	33	16.38	15.73	+ .65
7	16.58	16.81	- .23	80	16.55	15.86	+ .69	57	16.02	16.08	- .06	34	15.04	14.06	+ .98
8	16.62	15.88	+ .74	81	16.37	16.66	- .29	58	16.00	16.04	- .04	35	17.59	16.81	+ .78
9	15.64	14.90	+ .74	82	16.63	15.91	+ .72	59	14.51	13.22	+ 1.29	36	16.22	16.35	- .13
10	16.64	15.94	+ .70	83	14.69	13.60	+ 1.09	60	17.11	16.51	+ .60	37	16.13	16.23	- .10
11	15.99	15.20	+ .79	84	15.72	15.02	+ .70	61	16.52	15.80	+ .72	38	16.74	16.05	+ .89
12	16.05	16.14	- .09	85	15.92	15.98	- .06	62	17.14	16.49	+ .65	39	16.10	16.18	- .08
13	17.28	16.60	+ .68	86	15.95	16.12	- .17	63	17.17	16.52	+ .65	40	16.15	16.33	- .18
14	16.19	16.28	- .09	87	16.19	16.33	- .14	64	16.07	16.12	- .05	41	16.72	15.76	+ .96
15	15.97	16.03	- .06	88	16.75	16.02	+ .73	65	15.82	15.04	+ .78	42	15.68	14.73	+ .95
16	16.92	16.33	+ .59	89	17.19	16.59	+ .60	66	16.40	15.63	+ .77	43	15.22	14.13	+ 1.09
17	17.09	16.45	+ .64	90	15.17	14.19	+ .98	67	16.56	15.81	+ .75	44	15.93	15.98	- .05
18	17.07	16.36	+ .71	91	16.06	15.32	+ .74	68	16.79	16.07	+ .72	45	14.96	13.95	+ 1.01
19	17.14	16.52	+ .62	92	17.39	16.81	+ .58	69	14.47	13.19	+ 1.28	46	16.89	16.20	+ .69
20	15.46	14.75	+ .71	93	17.51	16.86	+ .65	70	15.72	14.92	+ .80	47	15.68	14.96	+ .72
21	17.18	16.62	+ .56	94	15.55	14.81	+ .74	71	15.82	15.04	+ .78	48	16.09	16.08	+ .01
22	15.78	14.94	+ .84	95	16.16	15.36	+ .80	72	15.91	15.14	+ .77	49	16.44	16.69	- .25
23	14.74	13.74	+ 1.00					73	17.43	16.73	+ .70	50	17.29	16.47	+ .82
Sector II															
24	17.12	16.41	+ .71	1	16.22	15.49	+ .73	75	16.23	15.50	+ .73	51	17.01	16.39	+ .62
25	16.31	16.57	- .26	2	17.22	16.66	+ .56	76	16.14	16.39	- .25	53	15.97	15.16	+ .81
26	17.58	16.98	+ .60	3	16.73	16.20	+ .53	77	16.66	15.92	+ .74	54	17.04	16.41	+ .63
27	15.82	15.23	+ .59	4	17.62	17.02	+ .60	78	16.37	16.71	- .34	55	14.44	13.17	+ 1.27
28	16.51	16.85	- .34	5	17.46	16.87	+ .59	79	15.91	15.89	+ .02	56	16.16	15.42	+ .74
29	17.44	16.98	+ .46	6	16.59	16.04	+ .55	80	16.00	16.11	- .11	57	16.02	16.07	- .05
30	15.70	14.79	+ .91	7	16.00	16.02	- .02	81	16.40	16.81	- .41	58	16.38	15.66	+ .72
31	16.53	15.87	+ .66	8	17.17	16.55	+ .62	82	16.06	16.22	- .16	59	16.21	16.30	- .09
32	16.48	15.76	+ .72	9	16.45	15.68	+ .77	83	17.42	16.84	+ .58	60	16.37	16.59	- .22
33	16.11	15.62	+ .49	10	15.12	14.18	+ .96	84	16.52	15.81	+ .71	61	15.78	14.95	+ .83
34	16.53	16.79	- .26	11	15.71	14.87	+ .84	85	15.93	16.10	- .17	62	16.94	16.23	+ .71
35	16.45	16.75	- .30	12	16.66	15.91	+ .75	86	16.74	16.12	+ .62	63	16.13	16.26	- .13
36	16.64	16.00	+ .64	13	17.19	16.87	+ .52	87	16.63	15.79	+ .84	64	17.28	16.74	+ .54
37	16.64	15.91	+ .73	14	16.04	16.10	- .06	88	16.76	16.15	+ .61	65	17.37	16.92	+ .45
38	17.42	16.75	+ .67	15	16.04	16.13	- .09	89	15.28	14.36	+ .92	66	16.44	16.62	- .18
39	16.35	15.77	+ .58	16	16.15	15.41	+ .74	90	16.02	16.15	- .13	67	17.26	16.56	+ .70
40	15.17	14.38	+ .79	17	16.63	15.94	+ .69	91	15.87	15.88	- .01	68	15.26	14.44	+ .82
41	16.41	15.75	+ .66	18	16.27	15.83	+ .64	92	16.09	16.19	- .10	69	16.83	16.12	+ .71
42	17.20	16.59	+ .61	19	14.91	13.89	+ 1.02	93	18.45	15.89	+ .56	70	16.59	15.93	+ .66
43	15.43	14.53	+ .90	20	16.97	16.37	+ .60	94	15.07	14.81	+ .26	71	14.92	14.17	+ .75
44	15.22	14.31	+ .91	21	16.83	16.23	+ .60	95	15.03	14.07	+ .96	72	16.69	16.03	+ .66
45	16.54	16.77	- .23	22	16.26	15.56	+ .70					73	16.03	15.96	+ .07
46	17.09	16.39	+ .70	23	15.83	15.95	- .12					74	17.52	16.75	+ .77
47	14.82	13.93	+ .89	24	15.76	15.17	+ .59	1	15.33	14.40	+ .93	75	17.38	16.83	+ .55
48	17.06	16.76	+ .30	25	16.67	16.87	- .20	2	16.20	15.73	+ .47	76	16.47	15.83	+ .64
49	17.56	16.94	+ .62	26	16.68	16.89	- .21	3	16.11	16.19	- .08	77	16.12	16.17	- .05
50	16.02	15.24	+ .78	27	15.07	14.16	+ .91	4	16.48	16.63	- .15	78	17.03	16.31	+ .72
51	16.04	16.34	- .30	28	16.24	16.59	- .35	5	17.7	17.3	+ .4	79	17.23	16.63	+ .60
52	17.09	16.38	+ .71	29	16.16	15.35	+ .81	6	17.29	16.73	+ .56	80	17.43	16.67	+ .76
53	16.51	15.83	+ .68	30	14.87	13.92	+ .95	7	16.17	16.30	- .13	81	14.93	14.00	+ .93
54	16.12	16.27	- .15	31	14.63	13.52	+ 1.11	8	16.47	15.71	+ .76	82	16.12	16.17	- .05
55	16.02	16.32	- .30	32	16.31	15.57	+ .74	9	17.33	16.37	+ .96	83	16.41	16.56	- .15
56	17.38	16.79	+ .59	33	15.02	14.08	+ .94	10	16.49	15.75	+ .74	84	16.11	15.36	+ .75
57	15.82	15.24	+ .58	34	16.22	15.81	+ .41	11	17.19	16.55	+ .64	85	15.83	15.19	+ .64
58	15.69	14.85	+ .84	35	16.51	16.75	- .24	12	16.64	15.87	+ .77	86	14.71	13.62	+ 1.09
59	14.99	15.19	- .20	36	17.22	16.56	+ .68	13	15.55	14.77	+ .78				
60	15.63	14.82	+ .81	37	15.11	14.51	+ .60	14	15.74	15.72	+ .02				
61	16.97	16.32	+ .65	38	16.67	15.97	+ .70	15	15.59	14.60	+ .99	1	17.12	16.53	+ .59
62	15.74	15.17	+ .57	39	16.41	16.66	- .25	16	17.30	16.72	+ .58	2	16.04	15.31	+ .73
63	15.30	14.87	+ .43	40	16.42	15.87	+ .75	17	16.97	16.49	+ .48	3	16.78	16.01	+ .77
64	16.75	16.06	+ .69	41	16.82	16.16	+ .66	18	15.98	15.08	+ .90	4	16.11	16.12	- .01
65	14.61	13.48	+ 1.13	42	16.93	16.30	+ .63	19	14.69	13.68	+ 1.01	5	16.08	16.09	- .01
66	16.33	15.62	+ .71	43	17.49	16.82	+ .67	20	15.50	14.70	+ .80	6	16.01	16.15	- .14
67	16.12	16.24	- .12	44	16.57	15.88	+ .69	21	16.96	16.27	+ .69	7	17.18	16.25	+ .93
68	16.63	15.97	+ .66	45	16.96	16.39	+ .57	22	17.29	16.62	+ .67	8	17.13	16.31	+ .82
69	16.88	16.26	+ .62	46	17.07	16.53	+ .54	23	15.94	15.17	+ .77	9	16.08	16.22	- .14
70	16.08	16.11	- .03	47	16.26	15.62	+ .64	24	16.62	16.68	- .06	10	15.88	15.00	+ .88
71	16.17	16.37	- .20	48	17.55	16.81	+ .74	25	16.09	15.37	+ .72	11	14.84	13.81	+ 1.03
72	17.13	16.60	+ .53	49	17.42	16.79	+ .63	26	16.63	15.83	+ .80	12			

M 2 (continued)															
No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.	No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.	No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.	No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.
Sector IV (cont.)												34	15.97	15.89	+.08
14	16.02	16.05	-.03	35	17.04	16.26	+.78	55	16.53	15.72	+.81	76	14.95	14.07	+.88
15	16.00	16.06	-.06	36	15.59	14.99	+.80	56	16.62	16.81	-.19	77	16.42	15.76	+.66
16	17.35	16.61	+.74	37	15.83	14.84	+.99	57	15.63	14.79	+.84	78	16.47	15.78	+.69
17	14.49	13.27	+.12	38	17.11	16.46	+.65	58	17.33	16.79	+.54	79	14.53	13.32	+.21
18	16.10	16.08	+.02	39	15.68	15.05	+.63	60	16.01	16.23	-.22	80	15.42	14.60	+.82
19	16.77	16.02	+.75	40	16.44	15.69	+.75	61	15.96	16.04	-.08	82	16.53	15.77	+.76
20	15.03	14.01	+.02	41	16.61	15.83	+.78	62	16.55	15.81	+.74	83	14.79	13.77	+1.02
21	15.87	15.11	+.76	42	14.90	14.09	+.81	63	16.81	16.05	+.76	84	16.41	16.67	-.26
22	15.19	14.30	+.89	43	14.78	13.92	+.86	64	15.97	16.11	-.14	85	16.52	16.77	-.25
23	16.14	16.30	-.16	44	16.21	15.39	+.82	65	16.21	16.46	-.25	86	16.69	15.96	+.73
24	16.47	15.70	+.77	45	16.62	15.98	+.64	66	17.28	16.33	+.95	87	16.16	16.28	-.12
25	17.09	16.48	+.61	46	16.18	16.37	-.19	67	16.07	16.33	-.26	88	16.22	15.75	+.47
26	16.53	15.72	+.81	47	16.66	16.07	+.59	68	16.24	15.54	+.70	89	16.07	16.12	-.05
27	16.46	16.69	-.23	48	15.97	16.18	-.21	69	16.51	15.79	+.72	90	15.64	15.18	+.46
28	17.07	16.43	+.64	49	17.21	16.58	+.63	70	16.08	16.17	-.09	91	17.14	16.37	+.77
29	16.23	15.88	+.35	50	16.14	16.29	-.15	71	16.91	16.08	+.83	92	17.55	16.62	+.93
30	17.45	16.68	+.77	51	16.22	16.38	-.16	72	16.11	16.11	-.00	93	15.26	14.34	+.92
31	15.93	15.14	+.79	52	15.54	14.72	+.82	73	16.68	15.91	+.77	94	16.22	16.43	-.21
32	16.73	16.05	+.68	53	15.68	14.93	+.75	74	16.85	15.88	+.97	95	15.97	15.39	+.58
33	15.78	15.81	-.03	54	16.35	15.62	+.73	75	15.43	14.56	+.87	96	14.51	14.61	-.10
M 5															
Sector I												54	16.17	15.33	+.84
1	14.82	14.08	+.74	55	14.43	13.60	+.83	25	15.46	14.97	+.49	79	16.41	15.74	+.67
2	14.73	13.68	+.05	56	15.70	14.84	+.86	26	16.64	15.91	+.73	80	15.03	14.18	+.85
3	14.80	13.97	+.83	57	15.46	14.57	+.89	27	15.13	14.12	+.01	81	15.00	15.06	-.06
4	14.37	13.25	+.12	58	14.25	13.15	+.10	28	15.43	14.47	+.96	82	15.05	14.92	+.13
5	14.93	13.95	+.98	59	15.43	14.41	+.02	29	16.07	15.28	+.79	83	15.68	14.89	+.79
6	15.55	15.76	-.21	60	15.01	15.03	-.02	30	15.12	14.99	+.13	84	15.87	15.03	+.84
7	15.61	15.56	+.05	61	14.42	13.35	+.07	31	15.57	14.59	+.98	85	13.83	12.33	+1.50
8	15.55	14.65	+.90	62	16.18	15.30	+.78	32	16.66	15.88	+.78	86	14.31	13.31	+1.00
9	16.03	15.13	+.90	63	15.21	15.27	-.06	33	15.85	15.02	+.83	87	15.62	15.78	-.16
10	15.60	14.82	+.78	64	15.82	14.99	+.83	34	15.74	14.88	+.86	88	15.88	15.10	+.78
11	15.59	15.66	-.07	65	15.83	15.05	+.78	35	16.75	15.97	+.78	89	15.45	14.94	+.51
12	14.81	14.06	+.75	66	15.32	15.41	-.09	36	16.24	15.43	+.81	90	16.38	15.69	+.69
13	15.38	14.90	+.48	67	14.63	13.90	+.73	37	15.53	15.62	-.09	91	16.27	15.75	+.52
14	14.16	12.98	+.18	68	13.87	12.32	+.55	38	15.33	14.92	+.41	92	15.08	15.11	-.03
15	14.80	14.13	+.67	69	15.35	15.45	-.10	39	15.23	14.18	+.05	93	15.12	14.23	+.89
16	15.19	15.21	-.02	70	15.22	15.22	-.00	40	16.69	16.04	+.65				
17	14.79	14.62	+.17	71	14.18	13.04	+.14	41	15.12	15.16	-.04				
18	17.12	16.46	+.66	72	15.33	15.40	-.07	42	16.10	15.33	+.77	1	16.23	15.49	+.74
19	16.36	15.58	+.78	73	16.03	15.27	+.76	43	16.41	15.67	+.74	2	15.09	15.07	+.02
20	13.74	12.43	+.31	74	14.86	13.97	+.89	44	15.50	15.04	+.46	3	13.83	12.40	+1.43
21	14.76	13.74	+.02	75	15.36	14.76	+.60	45	15.44	14.67	+.77	4	16.12	15.42	+.80
22	16.15	15.27	+.88	76	16.12	15.33	+.79	46	15.03	15.09	-.06	5	16.40	15.59	+.81
23	16.15	15.36	+.79	77	15.88	15.11	+.77	47	16.71	16.06	+.65	6	15.62	14.77	+.85
24	15.11	15.03	+.08	78	16.49	15.76	+.73	48	16.18	15.52	+.66	7	15.65	14.80	+.85
25	14.53	13.45	+.08	79	16.04	15.27	+.77	49	16.28	15.58	+.70	8	15.11	14.94	+.17
26	16.23	15.38	+.85	80	14.87	13.96	+.91	50	14.83	13.84	+.99	9	15.55	14.70	+.85
27	16.66	15.95	+.71	81	14.61	13.60	+.01	51	14.92	14.01	+.91	10	15.67	15.61	+.06
28	16.55	15.90	+.65					52	15.34	14.78	+.56	11	16.63	15.81	+.82
29	15.70	15.85	-.15					53	15.30	15.34	-.04	12	15.33	14.45	+.88
Sector II												54	15.10	14.16	+.94
30	16.37	15.69	+.68	1	15.22	14.37	+.85	55	16.11	15.18	+.93	13	15.67	14.81	+.86
31	15.57	14.78	+.79	2	16.52	15.68	+.84	56	15.00	14.87	+.13	14	16.27	15.51	+.76
32	16.87	16.22	+.65	3	16.43	15.59	+.84	57	15.32	14.85	+.47	15	15.08	14.20	+.88
33	16.63	15.95	+.68	4	16.76	16.06	+.70	58	15.78	14.92	+.86	16	14.88	14.19	+.69
34	15.28	15.44	-.16	5	16.01	15.16	+.85	59	14.34	13.21	+.13	17	15.03	15.01	+.02
35	14.95	13.98	+.97	6	16.82	15.93	+.89	60	15.72	14.92	+.80	18	14.28	13.31	+.97
36	15.44	14.62	+.82	7	15.77	14.79	+.98	61	14.39	13.40	+.99	19	15.03	14.05	+.98
37	15.76	16.03	-.27	8	16.47	15.67	+.80	62	15.14	15.07	+.07	20	16.56	15.87	+.69
38	16.56	15.88	+.68	9	13.84	12.35	+.49	63	15.27	14.64	+.63	21	16.42	15.68	+.74
39	14.19	13.07	+.12	10	16.29	15.40	+.89	64	15.78	14.97	+.81	22	15.17	15.18	-.01
40	15.55	14.80	+.75	11	15.45	14.87	+.58	65	16.70	15.91	+.79	23	16.25	15.52	+.73
41	15.19	15.22	-.03	12	15.62	14.75	+.87	66	16.27	15.46	+.81	24	14.07	13.51	+.56
42	16.29	15.60	+.69	13	15.78	14.92	+.86	67	16.08	15.31	+.77	25	16.41	15.64	+.77
43	14.85	13.94	+.91	14	15.05	14.99	+.06	68	15.69	14.90	+.79	26	14.86	14.12	+.74
44	14.83	14.85	-.02	15	15.27	15.32	-.05	69	15.08	14.13	+.95	27	15.02	14.25	+.77
45	15.47	14.97	+.50	16	15.73	14.95	+.78	70	16.67	15.97	+.70	28	15.66	14.77	+.89
46	16.67	15.86	+.81	17	15.81	15.03	+.78	71	16.89	16.31	+.58	29	15.77	14.93	+.84
47	15.64	14.87	+.77	18	15.18	14.38	+.90	72	15.21	15.24	-.03	30	15.63	14.71	+.92
48	15.10	14.95	+.15	19	17.10	16.45	+.65	73	16.03	15.14	+.89	31	15.16	14.13	+1.03
49	15.57	14.72	+.85	20	15.32	14.75	+.57	74	14.71	13.75	+.96	32	14.83	14.03	+.80
50	14.68	13.69	+.99	21	15.43	15.01	+.42	75	16.75	16.08	+.67	33	15.09	14.95	+.14
51	15.41	15.43	-.02	22	14.90	14									

M 5 (continued)															
No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.	No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.	No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.	No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.
Sector III (cont.)												72	15.69	15.70	-.01
38	15.13	15.07	+.06	73	15.13	14.99	+.14	21	15.50	15.61	-.11	55	16.17	15.36	+.81
39	16.09	15.30	+.79	74	16.78	16.07	+.71	22	15.59	14.74	+.85	56	14.21	13.09	+.12
40	16.31	15.56	+.75	75	16.49	15.63	+.86	23	15.74	14.89	+.85	57	15.23	15.31	-.08
41	15.75	14.94	+.81	76	16.54	15.72	+.82	24	15.83	15.05	+.78	58	15.39	15.57	-.18
42	16.11	15.33	+.78	77	16.12	15.34	+.78	25	16.46	15.65	+.81	59	13.92	12.69	+.23
43	15.15	15.12	+.03	78	14.01	12.61	+1.40	26	14.40	13.56	+.84	60	15.15	15.05	+.10
44	16.74	15.97	+.77	79	16.60	15.87	+.73	27	15.79	14.99	+.80	61	16.36	15.65	+.71
45	16.71	15.91	+.80	80	16.37	15.66	+.71	28	15.23	14.36	+.87	62	16.17	15.43	+.74
46	15.22	15.25	-.03	81	16.79	16.04	+.75	29	16.24	15.44	+.80	63	15.97	15.27	+.10
47	17.09	16.33	+.76	82	16.48	15.66	+.82	30	14.39	13.50	+.89	64	15.99	15.18	+.81
48	15.26	14.41	+.85	83	15.30	15.38	-.08	31	15.06	15.06	.00	65	14.38	13.90	+.48
49	15.70	14.83	+.87					32	15.75	14.91	+.84	66	16.30	15.66	+.64
50	14.05	12.88	+1.17					33	14.85	14.16	+.69	67	15.47	14.65	+.82
51	15.97	15.12	+.85	1	15.13	14.99	+.14	34	14.18	13.04	+1.14	68	15.07	15.09	-.02
52	14.90	13.93	+.97	2	15.45	15.48	-.03	35	15.65	14.82	+.83	69	15.54	15.08	+.46
53	14.39	13.50	+.89	3	15.67	14.78	+.89	36	14.83	13.88	+.95	70	16.04	15.29	+.75
54	16.86	16.12	+.74	4	14.95	13.90	+1.05	37	15.36	14.56	+.80	71	16.43	15.80	+.63
55	15.38	15.39	-.01	5	15.12	14.99	+.13	38	16.45	15.66	+.79	72	14.07	12.92	+1.15
56	14.31	13.32	+.99	6	15.12	14.17	+.95	40	15.51	14.60	+.91	74	14.51	13.47	+1.04
57	16.12	15.33	+.79	7	16.56	15.78	+.78	41	15.88	15.07	+.81	75	15.66	14.89	+.77
58	15.84	15.88	-.04	8	15.14	15.08	+.06	42	16.17	15.39	+.78	76	16.79	16.10	+.69
59	14.84	13.80	+1.04	9	15.21	14.71	+.50	43	15.81	15.02	+.79	77	16.60	15.89	+.71
60	15.78	14.87	+.91	10	15.15	14.29	+.86	44	15.71	14.92	+.79	78	14.08	13.64	+.44
61	16.19	15.37	+.82	11	14.93	14.04	+.89	45	16.59	15.80	+.79	79	15.96	15.26	+.70
62	15.18	14.22	+.96	12	14.48	13.41	+1.07	46	15.11	14.87	+.24	80	15.71	14.97	+.74
63	16.71	15.93	+.78	13	16.11	15.37	+.74	47	13.81	12.35	+1.46	81	13.69	12.12	+1.57
64	16.04	15.22	+.82	14	16.60	15.85	+.75	48	16.34	15.57	+.77	82	14.32	13.22	+1.10
65	15.25	15.21	+.04	15	16.66	15.87	+.79	49	14.18	13.09	+1.09	83	16.58	15.93	+.65
66	14.96	14.00	+.96	16	16.66	15.87	+.79	50	15.78	16.02	-.24	84	16.59	15.87	+.72
67	14.52	13.37	+1.15	17	15.57	15.69	-.12	51	15.38	14.88	+.50	85	16.06	15.36	+.70
68	15.45	15.38	+.07	18	16.63	15.83	+.80	52	16.69	16.07	+.62	86	15.37	14.94	+.41
69	15.09	14.93	+.16	19	13.96	12.58	+1.38	53	15.83	15.07	+.76	87	15.42	15.04	+.38
70	16.07	15.25	+.82	20	15.11	15.06	+.05	54	16.12	15.44	+.68	88	15.53	14.75	+.78
71	15.31	15.26	+.05												
M 10															
Sector I								Sector II							
1	16.39	15.69	+.70	40	15.54	15.47	+.07	1	16.45	15.59	+.86	41	16.76	15.58	+1.18
2	13.52	11.73	+1.79	41	16.48	15.56	+.92	2	15.06	13.86	+1.20	42	15.37	15.08	+.29
3	15.10	14.83	+.27	42	14.63	13.49	+1.14	3	15.48	15.20	+.28	43	15.83	14.71	+1.12
4	15.56	14.64	+.92	43	16.19	15.03	+1.16	4	16.08	15.17	+.91	44	15.40	15.18	+.22
5	16.79	15.73	+1.06	44	15.91	14.99	+.92	5	15.34	15.08	+.26	45	16.65	15.66	-.01
6	15.69	15.48	+.21	45	14.72	13.56	+1.16	6	15.58	14.54	+1.04	46	15.95	15.00	+.95
7	16.43	15.60	+.83	46	15.28	15.00	+.28	7	15.78	14.82	+.96	47	16.50	15.63	+.87
8	16.06	15.32	+.74	47	16.64	15.81	+.83	8	15.49	15.38	+.11	48	15.68	15.57	+.11
9	14.68	13.57	+1.11	48	15.10	13.91	+1.19	9	15.52	15.25	+.27	49	15.19	14.06	+1.13
10	15.76	14.78	+.98	49	15.35	14.38	+.97	10	16.47	15.67	+.80	50	13.85	12.39	+1.46
11	14.90	13.76	+1.14	50	15.96	15.60	+.36	11	15.29	14.10	+1.19	51	16.12	15.97	+.18
12	14.24	12.91	+1.33	51	13.86	12.53	+1.33	12	16.17	15.51	+.66	52	15.15	14.02	+1.13
13	15.53	14.51	+1.02	52	16.71	15.97	+.74	13	15.63	15.34	+.29	53	16.18	15.37	+.81
14	15.91	15.01	+.90	53	16.44	15.81	+.63	14	15.38	15.29	+.09	54	15.78	15.61	+.17
15	14.16	12.83	+1.33	54	15.79	14.95	+.84	15	15.14	14.03	+1.11	55	15.21	14.05	+1.16
16	15.64	15.26	+.38	55	15.74	14.77	+.97	16	15.78	14.80	+.98	56	15.45	15.18	+.27
17	15.59	15.33	+.26	56	14.70	13.57	+1.13	17	16.92	15.92	+1.00	57	15.33	14.29	+1.04
18	14.55	13.34	+1.21	57	15.92	15.06	+.86	18	16.64	15.72	+.92	58	16.01	15.18	+.83
19	16.10	15.27	+.83	58	16.29	15.49	+.80	19	16.10	15.36	+.74	59	16.92	15.94	-.02
20	16.83	15.81	+1.02	59	16.63	15.82	+.81	20	15.57	15.44	+.13	60	15.79	14.82	+.97
21	15.40	15.47	-.07	60	14.28	13.00	+1.28	21	15.25	14.93	+.32	61	16.79	15.89	+.90
22	15.31	15.11	+.20	61	14.26	12.99	+1.27	22	16.89	15.95	+.94	62	16.64	15.70	+.94
23	15.53	14.71	+.82	62	16.33	15.54	+.79	23	16.56	15.62	+.94	63	15.92	15.86	+.06
24	14.62	13.46	+1.16	63	14.38	13.58	+.80	24	13.62	11.94	+1.68	64	16.00	15.69	+.31
25	14.41	13.07	+1.34	64	16.22	15.45	+.77	25	15.58	15.43	+.15	65	15.66	14.65	+1.01
26	16.22	15.32	+.90	65	17.5:	16.01	+1.49:	26	16.04	15.81	+.23	66	16.79	15.86	+.93
27	16.04	15.92	+.12	66	16.22	15.33	+.89	27	17.01	16.10	+.91	67	15.34	14.26	+1.08
28	15.36	14.38	+.98	67	16.28	15.35	+.93	28	16.08	16.10	-.02	68	15.13	14.04	+1.09
29	15.08	14.87	+.21	68	16.68	15.82	+.86	29	17.05	15.91	+1.14	69	16.94	15.89	+1.05
30	15.70	14.64	+1.06	69	15.95	15.13	+.82	30	16.26	15.44	+.82	70	16.49	15.58	+.91
31	15.57	15.28	+.29	70	15.08	13.86	+1.22	31	16.00	15.69	+.31	71	15.24	14.62	+.62
32	13.87	13.78	+.09	71	15.88	15.49	+.09	32	14.98	14.05	+.93	72	12.61	11.85	+.76
33	13.36	13.47	-.11	72	16.51	15.86	+.65	33	15.74	15.42	+.32	73	16.06	16.06	.00
34	14.27	13.97	+.30	73	16.47	15.79	+.68	34	16.48	15.48	+1.00	74	15.03	13.94	+1.09
35	14.61	13.40	+1.21	74	15.67	14.81	+.86	35	15.98	15.02	+.96	75	16.45	15.59	+.86
36	16.29	15.43	+.86	75	16.62	15.79	+.83	36	14.51	13.24	+1.27	76	14.42	13.24	+1.18
37	16.44	15.66	+.78	76	15.87	14.99	+.88	37	14.44	13.44	+1.00	77	16.56	15.58	+.98

## M10 (continued)

No.	$m_{pg}$	$m_{pv}$	C.I.	No.	$m_{pg}$	$m_{pv}$	C.I.	No.	$m_{pg}$	$m_{pv}$	C.I.	No.	$m_{pg}$	$m_{pv}$	C.I.
<b>Sector II (cont.)</b>															
80	16.89	15.88	+1.01	26	15.73	14.77	+ .96	80	16.28	15.26	+1.02	33	15.68	15.49	+ .19
81	15.77	15.79	- .02	27	14.81	13.37	+1.24	81	14.98	13.68	+1.30	34	15.62	15.43	+ .19
82	15.05	14.77	+ .28	28	15.99	14.99	+1.00	82	15.30	15.16	+ .14	35	15.94	15.18	+ .76
83	15.17	14.53	+ .64	29	14.49	13.13	+1.36	83	15.71	14.69	+1.02	36	15.66	14.71	+ .95
84	15.90	15.78	+ .12	30	16.76	15.81	+ .95	84	16.61	15.38	+1.23	37	15.17	14.74	+ .43
85	15.23	14.18	+1.05	31	16.52	15.60	+ .92	85	14.13	12.76	+1.37	38	16.62	15.60	+1.02
86	15.94	14.99	+ .95	32	15.10	14.82	+ .28	86	16.97	15.87	+1.10	39	15.96	14.96	+1.00
87	15.59	14.48	+1.11	33	15.57	14.51	+1.06	87	16.76	15.87	+ .89	40	15.49	14.88	+ .61
88	15.82	15.63	+ .19	34	14.87	13.82	+1.05	88	15.67	14.72	+ .95	41	15.97	15.14	+ .83
89	var Nö.3			35	16.66	15.72	+ .94	89	16.95	16.00	+ .95	42	15.70	15.50	+ .20
90	15.50	14.37	+1.13	36	15.16	14.72	+ .44	90	15.76	14.77	+ .99	43	16.04	15.19	+ .85
91	16.38	15.60	+ .78	37	16.92	15.90	+1.02	91	14.28	13.09	+1.19	44	12.60	11.57	+1.03
92	16.82	15.90	+ .92	38	15.70	14.82	+1.08	92	15.05	14.07	+ .98	45	15.33	14.18	+1.15
93	17.03	15.89	+1.14	39	16.97	15.83	+1.14	93	14.26	12.87	+1.39	46	16.82	15.90	+ .92
94	15.54	14.46	+1.08	40	15.64	14.58	+1.06	94	15.45	14.63	+ .82	47	16.22	15.38	+ .84
95	15.61	14.62	+ .99	41	16.54	15.62	+ .92	95	15.92	14.91	+1.01	48	15.90	14.97	+ .93
96	16.63	15.77	+ .86	42	16.67	15.62	+1.05	96	16.58	15.57	+1.01	49	15.57	14.47	+1.10
97	16.45	15.62	+ .83	43	16.98	15.86	+1.12	97	14.02	12.64	+1.38	50	14.77	14.23	+ .54
98	14.96	13.78	+1.18	44	15.72	15.52	+ .20	98	15.87	15.03	+ .84	51	16.60	15.72	+ .88
99	16.87	16.05	+ .82	45	16.71	15.87	+ .84	99	15.12	14.18	+ .94	52	16.40	15.47	+ .93
100	16.68	15.86	+ .82	46	15.62	14.62	+1.00					53	15.71	14.83	+ .88
				47	15.40	14.38	+1.02					54	14.42	13.39	+1.03
								1	15.76	14.77	+ .99	55	16.61	15.61	+1.00
101	15.86	14.98	+ .88	48	14.52	14.37	+ .15	2	15.72	15.57	+ .15	56	16.38	15.50	+ .88
102	16.65	15.74	+ .91	49	16.38	15.57	+ .81	3	15.76	15.48	+ .28	57	15.18	15.22	- .04
103	16.04	15.05	+ .99	50	15.01	13.94	+1.07	4	14.86	13.67	+1.19	58	16.40	15.48	+ .92
104	14.71	13.43	+1.28	51	15.68	15.52	+ .16	5	15.77	14.64	+1.13	59	14.61	13.43	+1.18
105	14.25	12.82	+1.43	52	16.02	15.96	+ .06	6	15.49	14.43	+1.06	60	16.62	15.86	+ .76
106	15.21	15.06	+ .15	53	14.89	13.80	+1.09	7	14.80	13.99	+ .81	61	16.02	15.08	+ .94
				54	14.64	13.34	+1.30					62	16.61	15.89	+ .72
								8	14.31	13.19	+1.12	63	15.71	15.66	+ .05
								9	16.05	15.33	+ .72	64	14.77	13.87	+ .90
								10	15.98	15.07	+ .91	65	15.60	15.51	+ .09
								11	16.30	15.53	+ .77	66	15.31	15.06	+ .25
								12	16.18	15.34	+ .84	67	15.32	14.29	+1.03
								13	16.58	15.87	+ .71	68	15.42	14.38	+1.04
								14	15.92	14.94	+ .98	69	16.62	15.34	+1.28
								15	13.81	12.37	+1.44	70	15.52	15.39	+ .13
								16	15.28	15.07	+ .21	71	16.66	15.94	+ .72
								17	14.95	13.74	+1.21	72	15.56	15.49	+ .07
								18	15.87	15.65	+ .22	73	15.69	15.61	+ .08
								19	15.04	13.87	+1.17	74	16.59	15.77	+ .82
								20	15.10	13.90	+1.20	75	16.22	16.10:	+ .12:
								21	16.42	15.48	+ .94	76	15.72	14.81	+ .91
								22	16.48	15.50	+ .98	77	16.08	15.13	+ .95
								23	14.80	13.48	+1.32	78	15.88	14.78	+1.10
								24	16.71	15.76	+ .95	79	15.36	14.41	+ .95
								25	15.31	14.18	+1.13	80	15.05	15.06	- .01
								26	15.78	15.56	+ .22	81	16.57	15.81	+ .76
								27	16.31	15.36	+ .95	82	15.50	14.52	+ .98
								28	16.43	15.50	+ .93	83	15.53	15.53	.00
								29	15.52	14.40	+1.12	84	16.11	15.49	+ .62
								30	14.21	12.77	+1.44	85	14.83	13.72	+1.11
								31	16.57	15.67	+ .90	86	14.38	13.13	+1.25
								32	16.79	16.05	+ .74	87	15.01	13.94	+1.07

## M13

<b>Sector I</b>																
1	15.81	14.88	+ .93	21	14.93	15.07	- .14	*41	15.03	14.47	+ .56	61	16.13	15.62	+ .51	
*2	14.99	14.21	+ .78	22	15.58	14.92	+ .66	*42	13.95	12.85	+1.10	62	15.34	14.67	+ .67	
3	15.24	15.34	- .10	*23	14.02	13.07	+ .95	*43	14.48	14.18	+ .30	63	15.38	15.61	- .23	
4	15.65	15.75	- .10	*24	13.88	12.77	+1.11	44	15.47	14.87	+ .60	*64	14.99	14.94	+ .05	
5	16.10	15.33	+ .77	25	16.70	16.05:	+ .65:	*45	15.25	14.53	+ .72	*65	14.82	13.99	+ .83	
6	15.30	14.57	+ .73	26	15.99	15.41	+ .58	*46	15.22	14.61	+ .61	66	15.33	15.52	- .19	
7	15.10	15.20	- .10	27	15.63	14.90	+ .73	*47	15.09	15.24	- .15	67	15.09	14.36	+ .73	
8	15.47	15.72	- .25	28	15.41	14.57	+ .84	*48	13.51	11.91	+1.60	68	15.83	15.19	+ .64	
9	16.21	15.60	+ .61	29	15.68	15.08	+ .60	*49	14.74	13.89	+ .85	69	15.62	14.94	+ .68	
10	15.11	15.30	- .19	30	15.66	15.00	+ .66	*50	14.01	12.94	+1.07	70	15.56	14.98	+ .58	
11	15.31	15.62	- .31	31	15.04	14.33	+ .71	51	15.75	15.85	- .10	*71	14.72	14.66	+ .06	
*12	14.41	13.47	+ .94	32	15.64	14.92	+ .72	52	15.10	15.20	+ .10	72	15.24	14.49	+ .75	
*13	13.71	12.42	+1.29	*33	14.76	14.80	- .04	*53	14.12	13.26	+ .86	73	15.12	15.25	- .13	
14	15.99	15.35	+ .64	*34	15.27	14.56	+ .71	*54	14.41	13.67	+ .74	74	15.20	15.40	- .20	
*15	15.18	15.22	- .04	*35	14.74	13.91	+ .83	55	15.49	15.67	- .18	75	15.52	15.81	- .29	
*16	15.11	14.60	+ .51	36	15.42	14.74	+ .68	56	15.10	15.19	- .09	*76	14.58	13.74	+ .84	
*17	15.29	15.35	- .06	*37	14.85	13.99	+ .86	57	15.05	15.16	- .11	*77	14.16	13.08	+1.08	
*18	14.70	13.84	+ .86	38	15.28	14.53	+ .73	58	15.78	15.20	+ .58	*78	14.90	14.91	- .01	
*19	14.69	13.77	+ .92	*39	15.02	14.18	+ .84	59	15.34	15.53	- .19	79	15.11	15.30	- .19	

\* Four measures in each color.

M 13 (continued)															
No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.	No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.	No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.	No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.
Sector I															
80	16.06	15.50	+ .56	*55	15.05	15.14	- .09	22	15.88	15.25	+ .63	*18	14.57	13.69	+ .88
81	15.97	15.37	+ .60	56	15.98	15.33	+ .65	23	15.82	15.18	+ .64	*19	14.47	13.58	+ .89
82	16.31	15.68	+ .63	*57	13.78	12.62	+ .16	24	15.23	15.41	- .18	20	15.32	14.56	+ .76
*83	14.85	14.07	+ .78	58	15.92	15.37	+ .55	*25	14.78	14.16	+ .62	21	15.59	14.95	+ .64
*84	14.87	14.88	- .01	59	16.07	15.43	+ .64	*26	14.95	14.90	+ .05	*22	14.69	13.79	+ .90
*85	14.99	14.26	+ .73	60	15.70	15.04	+ .66	27	15.81	16.05	- .24	23	15.41	15.61	- .20
*86	14.06	13.01	+ 1.05	61	15.26	15.43	- .17	28	15.67	14.94	+ .73	*24	14.72	14.04	+ .68
87	15.41	14.73	+ .68	*62	15.44	15.58	- .14	29	15.36	14.69	+ .67	*25	13.53	12.01	+ 1.52
88	16.09	15.79	+ .30	63	15.22	14.50	+ .72	*30	14.12	13.06	+ 1.06	26	15.77	15.05	+ .72
*89	14.99	14.96	+ .03	64	15.69	15.08	+ .61	31	15.44	14.71	+ .73	*27	14.58	13.92	+ .66
90	15.14	15.26	- .12	*65	15.28	15.37	- .09	32	15.45	15.57	- .12	*28	13.99	12.88	+ 1.11
91	15.76	15.12	+ .64	66	15.65	14.91	+ .74	*33	12.64	12.98	- .34	29	15.51	14.83	+ .68
92	15.12	14.40	+ .72	*67	13.60	12.06	+ 1.54	34	15.09	14.35	+ .74	*30	15.25	15.32	- .07
93	15.59	14.97	+ .62	68	14.89	14.99	- .10	35	15.51	15.72	- .21	31	15.92	15.38	+ .54
94	15.47	15.61	- .14	69	15.64	14.99	+ .65	*36	14.73	13.86	+ .87	32	15.83	15.36	+ .47
				70	15.74	15.17	+ .57	*37	13.84	12.67	+ 1.17	33	15.39	14.88	+ .51
Sector II															
1	15.30	14.53	+ .77	*72	14.64	14.08	+ .56	39	15.03	14.30	+ .73	*35	13.99	13.03	+ .96
2	15.71	15.82	- .11	73	15.07	15.17	- .10	40	15.51	14.80	+ .71	*36	14.40	13.63	+ .77
3	16.04	15.32	+ .72	74	15.51	14.84	+ .67	*41	14.11	13.23	+ .88	*37	14.24	13.25	+ .99
4	15.28	14.56	+ .72	75	16.27	15.52	+ .75	42	15.14	15.25	- .11	*38	14.76	14.18	+ .58
5	15.26	15.37	- .11	*76	13.64	12.40	+ 1.24	*43	14.78	13.96	+ .82	*39	14.97	15.02	- .05
6	15.14	14.36	+ .78	77	15.30	15.46	- .16	44	15.40	14.61	+ .79	40	15.92	15.30	+ .62
7	15.02	14.27	+ .75	78	15.77	16.1 :	- .3 :	*45	14.39	13.48	+ .91	41	15.49	14.91	+ .58
8	15.67	15.79	- .12	79	15.43	14.76	+ .67	46	15.93	15.32	+ .61	42	15.19	15.32	- .13
* 9	14.87	14.03	+ .84	80	15.19	15.32	- .13	47	15.91	15.23	+ .68	*43	15.17	14.45	+ .72
10	15.08	14.26	+ .82	81	15.77	15.12	+ .65	48	16.19	15.62	+ .57	44	15.64	14.97	+ .67
11	16.43	15.46	+ .97	82	15.78	15.14	+ .64	*49	15.17	15.23	- .06	45	14.98	15.07	- .09
12	16.08	15.50	+ .58	83	14.84	14.98	- .14	*50	14.38	13.43	+ .95	*46	14.84	14.76	+ .08
13	15.97	15.36	+ .61	84	15.67	14.99	+ .68	*51	14.22	13.20	+ 1.02	*47	14.88	14.11	+ .77
*14	14.57	13.93	+ .64	85	15.35	14.85	+ .50	*52	13.74	12.53	+ 1.21	*48	Irr. var.		
15	15.88	15.26	+ .66	86	15.87	15.23	+ .64	53	15.27	15.45	- .18	*49	13.93	12.80	+ 1.13
16	15.67	15.01	+ .66	*87	14.18	13.19	+ .99	54	14.98	14.33	+ .65	50	15.76	15.14	+ .62
17	15.41	15.64	- .23	88	15.09	15.22	- .13	55	15.16	14.45	+ .71	51	15.24	14.45	+ .79
18	15.18	15.29	- .11	89	15.42	14.79	+ .63	*56	13.48	11.97	+ 1.51	*52	13.92	13.89	+ .03
19	15.33	15.43	- .10	*90	13.51	12.01	+ 1.50	57	15.53	14.82	+ .71	*53	13.70	12.54	+ 1.16
*20	15.30	15.39	- .09	91	15.90	15.39	+ .51	*58	15.02	15.06	- .04	*54	15.08	15.09	- .01
21	15.52	14.86	+ .66	92	15.58	14.95	+ .63	*59	13.70	12.46	+ 1.24	55	15.06	15.15	- .09
*22	15.26	15.36	- .10	93	15.56	15.05	+ .51	60	15.72	15.89	- .17	56	15.40	14.79	+ .61
*23	14.74	13.93	+ .81	*94	14.45	13.53	+ .92	61	15.44	14.72	+ .72	57	15.61	15.17	+ .44
24	16.02	15.46	+ .56	*95	14.62	13.74	+ .88	62	15.48	14.79	+ .69	58	16.07	15.42	+ .65
25	15.45	15.82	- .37	96	15.62	14.99	+ .63	*63	13.48	12.14	+ 1.34	*59	14.63	14.03	+ .60
*26	15.40	15.33	+ .07	97	15.06	15.17	- .11	*64	14.70	14.12	+ .58	*60	13.14	12.54	+ .60
27	15.33	14.64	+ .69	98	15.48	15.69	- .21	65	15.90	15.16	+ .74	*61	13.97	12.91	+ 1.06
*28	14.38	13.56	+ .82	99	15.17	15.27	- .10	66	15.24	15.52	- .28	62	15.68	15.15	+ .53
29	15.54	15.76	- .22	100	15.86	15.42	+ .44	67	16.12	15.47	+ .65	*63	15.11	15.21	- .10
*30	14.71	13.85	+ .86	*101	14.99	14.20	+ .79	68	14.99	15.12	- .13	64	15.55	14.86	+ .69
31	15.47	14.72	+ .75	102	15.56	14.87	+ .69	69	15.96	16.08:	- .12	65	15.97	15.33	+ .64
32	16.10	15.52	+ .58					70	15.19	15.29	- .10	*66	14.98	15.05	- .07
*33	13.79	12.54	+ 1.25					71	15.41	14.67	+ .74	67	15.29	14.56	+ .73
*34	13.59	12.20	+ 1.39	*1	15.03	15.13	- .10	*72	15.79	15.10	+ .89	68	15.19	14.46	+ .73
*35	14.38	13.43	+ .95	2	15.40	14.70	+ .70	73	13.56	12.25	+ 1.31	69	15.18	14.57	+ .61
36	15.74	15.17	+ .57	3	15.43	14.69	+ .74					70	15.71	15.07	+ .64
37	15.65	14.98	+ .67	4	15.28	15.43	- .15					*71	15.12	15.16	- .04
38	15.01	15.13	- .12	5	15.09	15.23	- .14	1	15.42	14.61	+ .81	72	15.90	15.34	+ .56
39	15.37	15.60	- .23	6	15.41	14.65	+ .76	*2	14.92	14.09	+ .83	73	15.12	15.24	- .12
*40	13.78	12.70	+ 1.08	*7	14.32	13.31	+ 1.01	3	15.43	14.69	+ .74	74	16.03	15.53	+ .50
*41	14.69	13.80	+ .89	8	15.07	15.17	- .10	*4	14.47	13.57	+ .90	75	15.07	15.17	- .10
*42	14.75	14.09	+ .86	*9	14.66	13.87	+ .79	5	15.07	14.27	+ .80	76	15.63	15.04	+ .59
43	15.29	15.50	- .21	10	15.43	14.77	+ .66	6	15.18	14.35	+ .83	77	15.25	14.57	+ .68
44	16.03	15.49	+ .54	*11	15.00	15.03	- .03	7	15.40	14.67	+ .73	*78	14.84	14.03	+ .81
45	16.07	15.47	+ .60	*12	14.37	13.49	+ .88	8	15.34	14.48	+ .86	79	15.88	15.29	+ .59
46	15.36	15.52	- .16	13	15.53	15.73	- .20	9	15.94	15.36	+ .58	*80	15.01	14.23	+ .78
47	15.03	15.24	- .21	14	15.87	15.34	+ .53	10	15.02	15.16	- .14	81	15.18	15.32	- .14
*48	13.67	13.69	- .02	15	15.74	15.11	+ .63	11	16.10	15.52	+ .58	82	16.07	15.53	+ .54
*49	14.63	13.81	+ .82	*16	15.09	15.21	- .12	12	15.57	15.02	+ .55	*83	15.01	15.08	- .07
*50	14.82	13.98	+ .84	17	15.76	15.92	- .16	13	15.26	14.50	+ .76	84	15.33	14.70	+ .63
*51	14.97	14.15	+ .82	*18	13.87	12.70	+ 1.17	14	15.66	15.79	- .13	85	15.87	15.48	+ .39
52	15.08	14.41	+ .67	19	15.89	15.34	+ .55	*15	13.86	12.88	+ .98	86	15.79	15.10	+ .69
53	15.63	15.06	+ .57	20	16.21	15.65	+ .56	*16	14.89	14.09	+ .80	87	16.19	15.58	+ .61
M 15															
Sector I															
1	15.82	15.85	- .03	5	15.94	15.33	+ .61	9	15.78	15.78	.00	13	16.97	16.43	

M 15 (continued)											
No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.	No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.	No.	m <sub>pg</sub>	m <sub>pv</sub>	C.I.
			Sector I	15	15.21	14.63	+ .58	2	16.92	16.39	+ .53
16	16.59	15.96	+ .63	16	14.33	13.38	+ .95	3	15.81	15.77	+ .04
17	16.22	15.62	+ .60	17	16.37	15.53	+ .84	4	15.79	15.06	+ .73
18	15.69	15.15	+ .54	18	16.96	16.33	+ .63	5	16.18	15.92	+ .26
19	17.07	16.58	+ .49	19	17.39	16.67	+ .72	6	16.63	15.99	+ .64
20	15.63	15.03	+ .60	20	16.32	16.54	- .22	7	16.44	16.54	- .10
21	16.19	15.55	+ .64	21	16.75	16.25	+ .50	8	14.74	13.91	+ .83
22	15.64	15.35	+ .29	22	16.81	16.27	+ .54	9	15.43	14.75	+ .68
23	14.93	14.03	+ .90	23	15.74	15.80	- .06	10	15.87	15.24	+ .63
24	16.00	15.74	+ .26	24	15.92	15.60	+ .32	11	16.31	15.72	+ .59
25	17.17	16.52	+ .65	25	16.67	16.84	- .17	12	15.11	14.26	+ .85
26	17.32	16.78	+ .54	26	16.07	15.85	+ .22	13	16.05	15.28	+ .77
27	17.36	16.72	- .36	27	16.53	16.74	- .21	14	14.29	13.74	+ .55
28	15.28	14.38	+ .90	28	16.64	16.10	+ .54	15	15.74	15.87	- .13
29	16.49	16.39	+ .10	29	14.20	13.07	+1.13	16	16.63	16.87	- .24
30	16.03	15.22	+ .81	30	14.32	13.24	+1.08	17	17.65	17.00	+ .65
31	16.49	16.06	+ .43	31	14.20	13.09	+1.11	18	16.75	16.78	- .03
32	16.87	16.42	+ .45	32	16.55	16.35	+ .20	19	15.63	14.89	+ .74
33	15.79	15.07	+ .72	33	16.23	15.66	+ .57	20	16.83	16.26	+ .57
34	15.87	15.97	- .10	34	16.42	16.62	- .20	21	16.47	16.65	- .18
35	17.39	16.76	+ .63	35	17.43	16.80	+ .63	22	17.60	16.87	+ .73
36	17.42	16.89	+ .53	36	15.88	15.98	- .10	23	17.32	16.72	+ .60
37	17.6	16.93	+ .67:	37	16.27	15.74	+ .53	24	17.53	17.05	+ .48
38	14.83	14.09	+ .74	38	16.93	16.48	+ .45	25	17.55	16.82	+ .73
39	17.03	16.53	+ .50	39	16.97	16.42	+ .55	26	15.10	14.19	+ .91
40	17.29	16.80	+ .49	40	15.10	14.28	+ .82	27	14.25	13.17	+1.08
41	14.89	13.98	+ .91	41	15.92	15.54	+ .38	28	15.96	15.63	+ .33
42	16.27	16.48	- .21	42	14.14	13.04	+1.10	29	17.10	16.56	+ .54
43	14.58	13.73	+ .85	43	15.32	14.72	+ .60	30	16.58	16.08	+ .50
44	17.06	16.52	+ .54	44	15.06	14.46	+ .60	31	17.65	16.98	+ .67
45	16.92	17.02	- .10	45	16.14	15.58	+ .56	32	15.55	15.02	+ .53
46	16.95	16.42	+ .53	46	16.60	16.16	+ .44	33	14.76	13.96	+ .80
47	16.64	16.78	- .14	47	17.08	16.51	+ .57	34	13.01	12.56	+ .45
48	15.86	15.21	+ .65	48	16.37	15.82	+ .55	35	17.40	16.78	+ .62
49	16.65	15.99	+ .66	49	15.71	15.03	+ .68	36	17.28	16.83	+ .45
50	14.43	13.40	+1.03	50	16.31	15.72	+ .59	37	15.59	14.92	+ .67
51	15.81	15.82	- .01	51	15.62	15.03	+ .59	38	16.25	15.69	+ .56
52	15.86	16.05	- .19	52	17.47	16.92	+ .55	39	17.06	16.55	+ .51
53	16.43	15.77	+ .66	53	17.04	16.47	+ .57	40	17.41	16.81	+ .60
54	15.78	15.82	- .04	54	15.74	15.89	- .15	41	15.91	15.34	+ .57
55	15.87	16.04	- .17	55	16.55	16.10	+ .45	42	17.35	16.72	+ .63
56	14.72	13.83	+ .89	56	15.88	16.02	- .14	43	15.77	15.87	- .10
57	15.99	14.73	+1.26	57	17.16	16.68	+ .48	44	15.63	14.97	+ .66
58	15.79	15.92	- .13	58	16.02	15.49	+ .53	45	16.37	15.83	+ .54
59	15.93	15.30	+ .63	59	15.85	15.95	- .10	46	15.30	14.72	+ .58
60	17.14	16.62	+ .52	60	16.63	16.16	+ .47	47	16.88	16.26	+ .62
61	16.32	15.76	+ .56	61	16.27	16.75	- .48	48	14.43	13.39	+1.04
62	15.05	14.39	+ .66	62	17.18	16.82	+ .36	49	17.55	16.82	+ .73
63	14.96	14.12	+ .84	63	17.13	16.69	+ .44	50	16.58	16.00	+ .58
64	17.27	16.72	+ .55	64	14.42	13.39	+1.03	51	16.12	15.56	+ .56
65	15.71	15.04	+ .67	65	17.12	16.52	+ .60	52	15.72	15.76	- .04
66	15.81	15.27	+ .54	66	17.46	16.95	+ .51	53	15.87	16.04	- .17
67	16.54	17.03	- .49	67	16.70	16.19	+ .51	54	15.89	15.30	+ .59
68	16.48	16.14	+ .34	68	16.65	16.13	+ .52	55	17.18	16.71	+ .47
69	15.93	15.35	+ .58	69	17.01	16.43	+ .58	56	17.23	16.65	+ .58
70	16.52	16.81	- .29	70	17.27	16.83	+ .44	57	17.44	16.94	+ .50
71	15.87	16.04	- .17	71	17.03	16.42	+ .61	58	16.44	16.75	- .31
72	15.62	15.12	+ .60	72	16.48	15.99	+ .49	59	14.88	14.09	+ .79
73	16.39	16.75	- .36	73	15.78	15.94	- .16	60	16.86	16.40	+ .46
74	14.73	13.88	+ .85	74	15.88	16.00	- .12	61	15.90	15.31	+ .59
75	16.25	15.75	+ .50	75	14.08	12.87	+1.21	62	16.50	16.03	+ .47
			Sector II	76	16.13	15.67	+ .46	63	15.84	16.01	- .17
				77	15.61	15.08	+ .53	64	16.26	15.67	+ .59
1	15.93	15.35	+ .58	78	16.28	15.83	+ .45	65	17.55	16.95	+ .60
2	16.41	15.75	+ .66	79	16.50	16.61	- .11	66	17.27	16.77	+ .50
3	16.81	16.25	+ .56	80	17.45	16.80	+ .65	67	15.86	15.97	- .11
4	17.30	16.81	+ .49	81	17.34	16.87	+ .47	68	17.23	16.71	+ .52
5	17.40	16.92	+ .48	82	16.78	16.27	+ .51	69	16.08	15.51	+ .57
6	17.31	16.93	+ .38	83	16.10	15.53	+ .47	70	15.09	14.30	+ .79
7	15.58	15.11	+ .47	84	17.02	16.56	+ .46	71	15.82	15.55	+ .27
8	15.38	14.78	+ .60	85	17.55	16.95	+ .60	72	17.19	16.65	+ .54
9	15.12	14.22	+ .90	86	17.38	16.69	+ .69	73	17.56	16.91	+ .64
10	15.90	15.91	- .01	87	17.48	16.83	+ .65	74	16.20	15.75	+ .45
11	15.72	15.80	- .08	88	17.65	17.1	+ .55:	75	16.49	15.88	+ .61
12	17.51	16.92	+ .59					76	16.79	16.82	+ .97
13	16.89	16.26	+ .63					77	15.34	14.54	+ .80
14	15.60	15.33	+ .27	1	17.65	17.10	+ .55				

to a variable which now seems irregular although it was originally listed with a period of 106 days.

No RR Lyrae stars were measured and the modulus of the cluster was estimated solely from the position of the gap.

**3.3 M<sub>13</sub>.** (Figure 8.) The diagram of this cluster deviates most from the pattern of the rest of the clusters investigated here. The most unusual feature is the ill-defined gap in the horizontal sequence and the large slope of the blue end of that sequence. The measures here, as well as measures of fainter stars by Savedoff, indicate that this blue branch extends to fainter magnitudes than in any of the other clusters investigated.

Because of the difficulty of identifying the center of the gap, only RR Lyrae stars were used to determine the zero point in this cluster. Only two bona fide RR Lyrae stars could be measured; an additional pair, undoubtedly RR Lyrae, are too close together and too near the center of the cluster. Two-color light curves for the measurable pair are shown in Figures 14 and 15; data for them are collected in Table IIa. Since they have standard light curves for their respective periods and since they both give very nearly the same value for the modulus of the cluster, the zero point is considered to be accurate within  $\pm 0.1$  mag. This zero point is confirmed by the fact that it gives, within a few hundredths of a magnitude, the same absolute magnitude for a 1.4-day period cepheid in M<sub>13</sub> as that of a nearly identical cepheid in M<sub>15</sub>. The same is true of a 2.2-day period in M<sub>13</sub> which has an almost exact counterpart in  $\omega$  Centauri. The zero point, as judged by the mean position of the two RR Lyrae stars, gives a reddening of 0.16 mag.

This particular cluster was selected as a test case in which to determine how far the bifurcation suggested in most of the giant sequences could be traced. Stars brighter than  $m_{\text{pv}} = 14.4$  were measured on four plates, in both colors,

taken with the 60-inch telescope. The probable errors for the mean magnitudes marked with an asterisk in Table I then are of the order of 0.01 mag. or less. It is now seen that in this cluster the bifurcation can be traced nearly to the end of the giant sequence although the mean separation becomes very small toward the bright end of the sequence.

If the red side of the giant branch is considered as a separate sequence, it appears that very narrow sequences can exist in such diagrams. The total spread, in color index, is only about 0.1 mag. for this feature. This, however, is still larger than the accidental errors of measurement. Judging from the diameter of the cluster (the largest of the seven clusters) the back-to-front magnitude difference cannot exceed 0.02 mag. If there is no variable obscuration over the area measured, and this already must be less than 0.1 mag. in reddening, then the width of the giant sequence derived for M<sub>13</sub> must be considered as its intrinsic width.

The radial velocity of the very blue star at  $m_{\text{pv}} = 12.98$  shows it to be a member of the cluster (Greenstein and Münch, unpublished results).

**3.4 M<sub>10</sub>.** (Figure 9.) Since only one 60-inch exposure in each color was measured, the accidental error of the plotted points is slightly greater than that for any of the other diagrams. The diagram of this cluster most resembles that of M<sub>2</sub>.

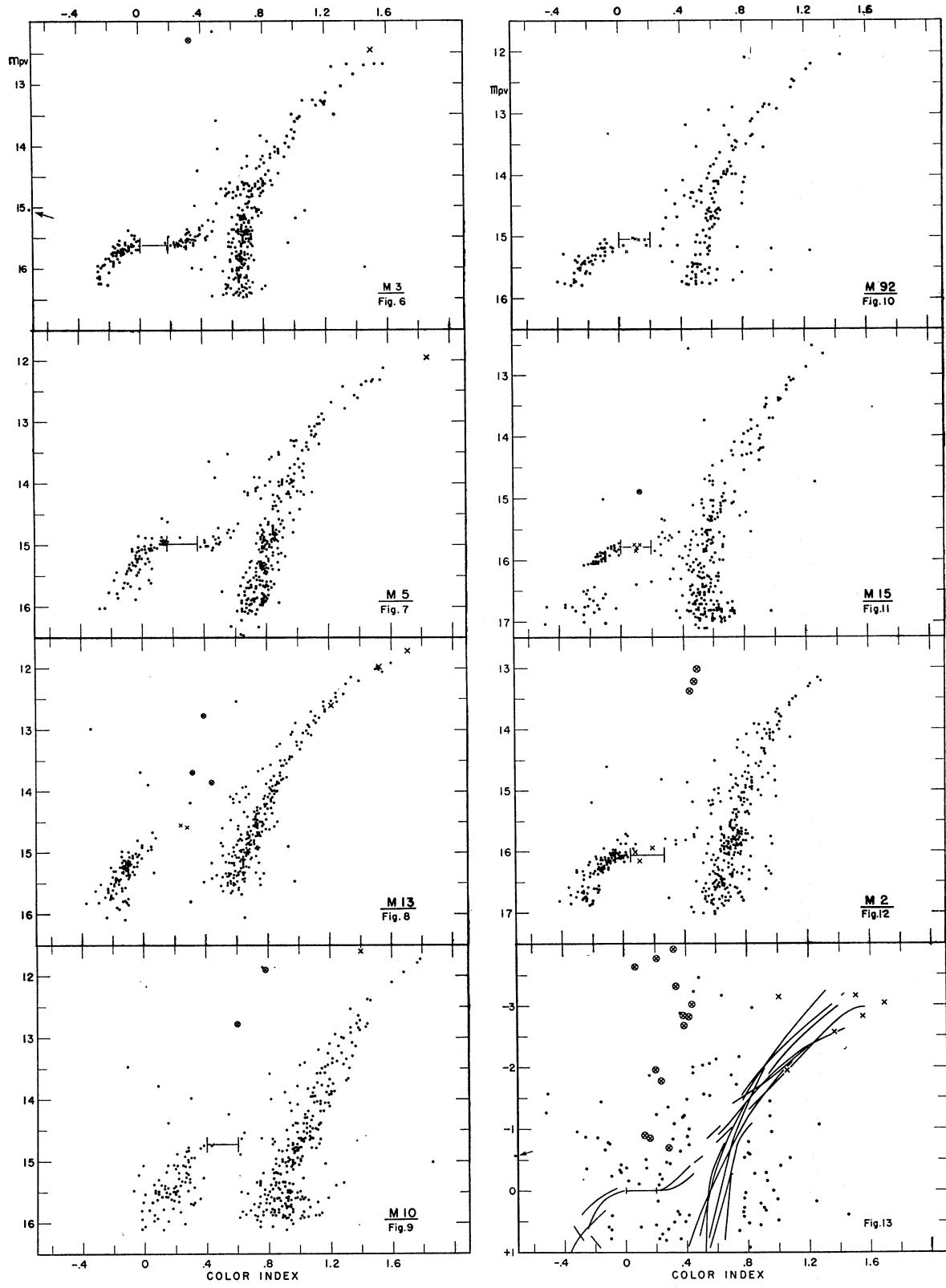
The cluster is obviously highly reddened, the position of the gap giving 0.4 mag. reddening. Since this would indicate between one and two magnitudes absorption, the real distance modulus would be around  $m_{\text{pv}} = 13$  mag., making this cluster easily one of the closest known.

M<sub>10</sub> is the only cluster investigated here which has no known RR Lyrae variables. Such variables have been looked for by Oosterhoff, Mrs. Hogg, and the writer, among others. Despite the

TABLE II. RR LYRAE STARS

Hogg No.	Type	Period		$m_{\text{pg}}$	Max	Min	Mean*		$m_{\text{pv}}$	Max	Min	Mean*	Observed epoch of maximum J.D. 2434000. +	
a. M <sub>13</sub>														
7	c	.2388			14.54	15.00	14.79			14.36	14.67	14.55		157.733
8	a	.750306			14.35	15.30	14.87			14.20	14.90	14.59		156.709
b. M <sub>2</sub>														
3	a	.619705			15.30	16.70	16.09			15.43	16.40	16.01		272.83 $\pm$ .03
7	b	.594857			15.45	16.84	16.15			15.50	16.44	15.96		305.60 $\pm$ .06
9	a	.60981			15.52	16.86	16.28			15.62	16.66	16.17		272.67 $\pm$ .03

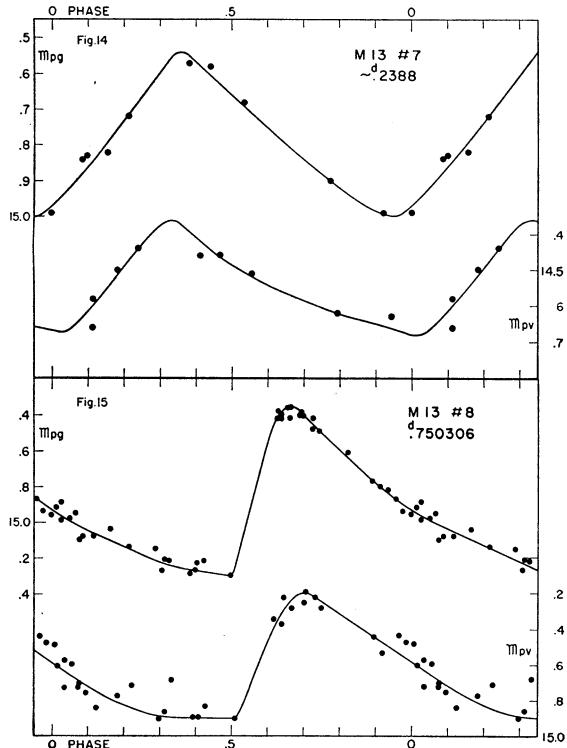
\* Magnitude at mean intensity.



Figures 6-12. Legend on opposite page.

fact that the cluster appears loose enough to see into the very central regions, only the 7.9-day and 18.8-day cepheids and the irregular variable have been discovered.

The population of the red side of the horizontal sequence is sparse, again correlating with the lack of RR Lyrae stars. Although the number of stars in the blue branch decreases very close to the faint end, no definite cut-off was observed to the  $M_{\text{pv}} = +1.0$  mag. limit measured.



Figures 14 and 15. Light curves for RR Lyrae stars M13, No. 7 and No. 8.

3.5 *M<sub>92</sub>*. (Figure 10.) The diagram for this cluster contains fewer stars than the rest of the diagrams, only 177 in the 124'' to 475'' annulus in this magnitude range. As a consequence the RR Lyrae stars plotted in the gap give a better modulus determination than the blue end of the gap.

The diagram generally resembles that of M<sub>2</sub> and M<sub>10</sub> and the bifurcation of the giant branch seems to be well marked. Measures from more

central regions (Arp, Baum, and Sandage 1953) mark the terminus of the giant branch at C.I. = +1.4 mag. The blue branch stops about 1 mag. below the gap.

3.6 *M<sub>15</sub>*. (Figure 11.) This diagram represents the most extreme case of the M<sub>2</sub>, M<sub>92</sub> pattern. The cluster is very centrally condensed but by measuring as close to the center as background effects allowed, a diagram almost completely free from field stars was obtained.

The horizontal sequence in this cluster is very unusual. There is a moderate population on the red side of the gap which is correlated with the fairly large number of RR Lyrae stars known in this cluster. The blue end of the sequence, however, is very narrow. The measures in this diagram are from one exposure with the 100-inch in each color and hence the accuracy is comparable with that in the rest of the diagrams. This suggests that the observed width of the blue sequence in other diagrams is the intrinsic width in those particular clusters and not due to the errors of measurement.

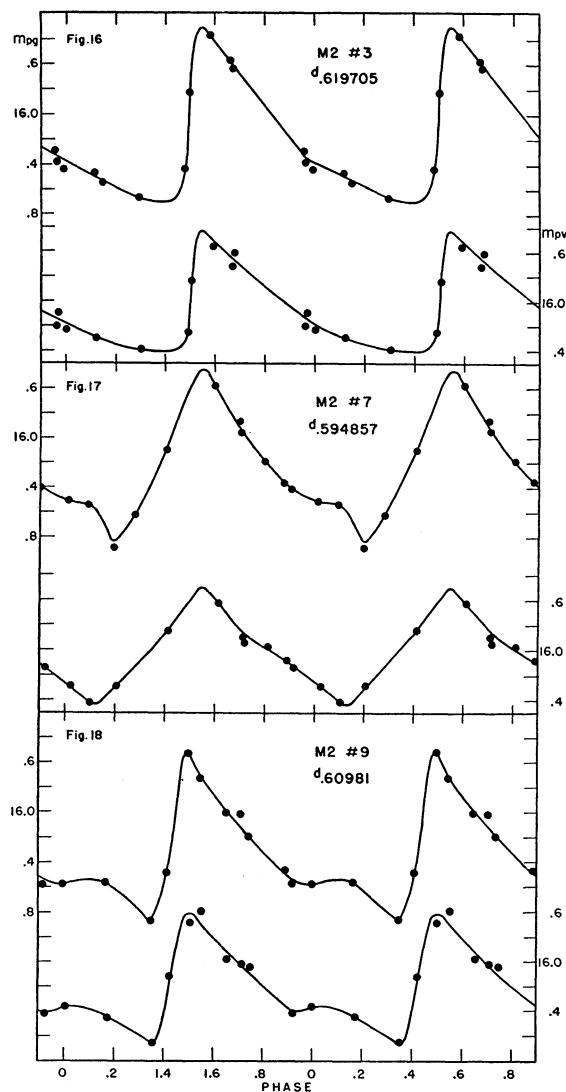
The small, narrow blue branch stops abruptly 0.3 mag. below the gap. At about a magnitude below the gap more stars in this sequence are encountered over a range of about half a magnitude. Search for fainter stars in this region seems to yield nothing (John Schopp, private communication). Present indications, then, are that the faint blue-sequence stars form an isolated clump at about  $m_{\text{pv}} = 16.75$  mag.

Although the position of the gap is exceedingly well determined by the narrow blue sequence, a few RR Lyrae stars were sampled to see whether they would fall in the gap. The light curves are not shown since only about seven or eight points were obtained on each curve. However, for the four RR Lyrae stars that had points well distributed over their cycles, namely, Nos. 9, 32, 53, and 66, this gave mean magnitudes with sufficient accuracy to plot them in the gap with the small crosses as indicated in the diagram. These RR Lyrae stars seem to have rather small amplitudes, although the number sampled is too small to show any general conclusion.

3.7 *M<sub>2</sub>*. (Figure 12.) Since there is about 0.05 mag. reddening indicated for this cluster,

Figures 6-12. Color-magnitude diagrams for globular clusters. Circled crosses represent cepheids, large crosses long-period or irregular variables, and small crosses RR Lyrae stars. Figure 6: 15-day cepheid; 103-day long-period variable. Figure 7: Cepheids too bright to be included; one irregular variable. Figure 8: 97-day long-period; one irregular variable. Figure 9: 8- and 18-day cepheids; one irregular variable. Figure 11: 1.4-day cepheid. Figure 12: 15-, 17-, and 19-day cepheids.

Figure 13. Superposition of all seven color-magnitude diagrams. One line may represent sequence from several clusters in same region of diagram. Circled crosses indicate cepheids; crosses, long-period and irregular variables. Remaining points represent stars falling off sequences in individual diagrams.



Figures 16, 17, and 18. Light curves for RR Lyrae stars M<sub>2</sub>, No. 3, 7, and 9.

the giant branch extends only to a normal color index of +1.2 mag. which is the bluest limit of any of the giant branches investigated here.

The blue portion of the horizontal sequence is the most heavily populated of any of the seven clusters, even stronger than in M<sub>3</sub>. Its faint limit seems to be at  $M_v = +0.8$  mag. although this is too near the sample limit to be certain. There is slight evidence for a "clump" near this limit as in M<sub>15</sub>.

The mean position of the three RR Lyrae cepheids confirms within a few hundredths of a magnitude the position of the gap as drawn. The moderate population on the red side of the gap

which is correlated with the dozen RR Lyrae stars in the cluster, however, is at a slightly higher luminosity than in most clusters. This may signify that the gap is not horizontal in this cluster but runs at a slight angle to join the few stars to the red of the gap. Accurate mean magnitudes of all the RR Lyrae stars present would be required, however, to decide this.

Of the 12 stars listed as RR Lyrae, three were far enough from the cluster so that they could be measured on 60-inch plates at this apparent magnitude without incurring background effects. The light curves for these three are shown in Figures 16, 17, 18; data for them are collected in Table IIb.

#### 4.0 General Properties.

**4.1 General form of the diagrams.** Comparison of the whole group of diagrams reveals the following pattern. At the bright limit the giant branch is very narrow. As the giant branch progresses toward fainter magnitudes, a minority of stars split off to the blue, increasing their separation from the main giant branch. The blue side of this so-called bifurcation of the giant sequence seems to continue toward the red end of the horizontal sequence while the remaining stars turn down to become the vertical part of the giant branch. There is always a strict gap, however, at about  $M_{pv} = -0.6$  mag., between the blue part of the giant sequence and the red side of the horizontal sequence. Also, at the point where the blue side of the giant branch stops, about  $M_{pv} = -1.0$  mag., there is a fill-in of stars between it and the main part of the giant branch. This makes for a slight increase in star density at this point in the diagrams. In this region also occur a few scattered stars 0.2 and 0.3 mag. to the blue side of the sequences which could, conceivably, be associated with the behavior of the sequences at this point.

Further down, at about the magnitude of the gap, inspection of either the diagrams or Table III reveals a slight concentration or clumping in the vertical giant branch. Although this is not a conspicuous feature, the same pattern is repeated in all the clusters.

It is difficult to specify the exact nature of these details, much less their interpretation, but they do hint that the diagrams may be more complicated than the first impression of a simple inverted "y" form.

**4.2 Luminosity functions.** Table III gives the numbers of stars in intervals of absolute photo-

TABLE III. NUMBER OF STARS IN DIAGRAMS IN INTERVALS OF PHOTOVISUAL MAGNITUDE \*

$M_{pv}$	$M_3$ 100''-600''	$M_5$ 115''-275''	$M_{13}$ 100''-300''	$M_{10}$ 80''-380''	$M_{92}$ 124''-475''	$M_{15}$ 70''-150''	$M_2$ 100''-290''
-3.4 to -3.2				I		I	2
-3.2 to -3.0	3.5			2	2	I	
-3.0 to -2.8	2.5	I		I	2	3	4
-2.8 to -2.6	I	6	6	I	2	2	2
-2.6 to -2.4	6	2	3	2	I	5	4
-2.4 to -2.2	4	3	3	I	5	2	7
-2.2 to -2.0	4	6	I	5	4	5	10
-2.0 to -1.8	5	4	5	7	3	6	7
-1.8 to -1.6	8	9	I	8	5	7	6
-1.6 to -1.4	8	7	7	8	4	4	I
-1.4 to -1.2	8	7	3	2	9	3	0 2 14
-1.2 to -1.0	24	19	9	I	10	14	5
-1.0 to -.8	16	16	2	10	I	12	I
-.8 to -.6	I I 13	8	15	I2	8	I3	I1
-.6 to -.4	0 2 19	2 I 8	I 17	2 II	0 2 16	0 2 7	I 2 20
-.4 to -.2	3 14 29	5 6 20	I5	0 I II	0 I 5	0 7 I7	5 5 33
-.2 to 0.0	25 35 19	21 12 26	I	25	5 2 32	4 I 6	15 2 15 30 19
0.0 to +.2	23 26	I5	2	26	8 I 15	I5 I 6	20 I 14 26 15
+ .2 to .4	8 19	I2	25	I3 27	I3 15	I3 II	18 7 24
.4 to .6	6 25	8 22	30 22	I4	I5	8 10	2 I5 15 24
.6 to .7	16	3 8	I4 16	I6	I5	4: 5:	3 I2 7 I2
.7 to .8	7	2	I2 9	I1	8	18	3 9 3 5:
.8 to .9			I5 6	10	6	I3	9 I5 4:
.9 to 1.0		2 8	8	5	I5		3 21
Total brighter than $M_{pv} = 0.7$	66 52 256 374	66 19 212 297	65 0 222 287	64 4 189 256	44 5 128 177	42 12 168 222	93 9 247 349

\* Stars far to the red of sequences, and variables, omitted.

visual magnitude in the diagrams of each of the seven clusters. In each column, numbers on the left refer to stars bluer than C.I. = 0.0 mag., in the middle to stars redder than this but not so red as to be included in the giant sequence, and on the right to stars in the giant sequence. The sample in all clusters within the annulus measured is complete to at least  $M_{pv} = +0.7$  mag. The total number of stars brighter than this limit is given at the bottom of each column.

Table III may be used to form relative luminosity functions for the clusters or to compare percentage populations of various features. Differences in the population of the horizontal sequence from cluster to cluster are most noticeable. The horizontal sequence in clusters like  $M_3$  and  $M_5$  has a sharp maximum in its luminosity function at the magnitude of the gap. In  $M_{13}$  and  $M_{10}$ , however, the function increases towards fainter magnitudes.

Differences from cluster to cluster between the relative luminosity functions of the giant branches also seem to be present but they approach the statistical significance of the sample and do not

appear to be correlated with other properties of the clusters.

The differences in the blue-branch luminosity functions, however, are so large as to make the total relative luminosity function quite different in form in the various clusters. Therefore it does not seem possible to estimate total comparative populations in globular clusters by making the luminosity functions from -3 to +1 agree by a scale factor.

4.3 *Integrated absolute magnitudes.* By applying the apparent distance moduli in Table V to the integrated apparent magnitudes given by Christie (1940), the absolute photographic magnitudes of the clusters were obtained. They are listed in the last column of Table VI. The luminosities of the seven globular clusters range from  $M_{pg} = -7.6$  to  $-8.9$  mag. The absolute magnitudes of the brightest clusters known,  $\omega$  Centauri and 47 Tucanae, are about  $M_{pg} = -9.5$  mag.

The relative luminosity functions of Table III enable an estimate to be made of how stars at different magnitude levels contribute to this integrated brightness. In photovisual wave lengths,

TABLE IV. NUMBER OF STARS ACROSS GAP IN INTERVALS OF COLOR INDEX

Color index	$M_3$	$M_5$	$M_{13}$	$M_{10}$	$M_{15}$	$M_2$
-0.2 to 0.0	51	48	I2	41	33	58
0.0 to +0.2*	105	41	I	0	27	7
+0.2 to +0.4	44	18	0	4	I2	3

\* RR Lyrae stars.

about 50 per cent of the total light of M<sub>3</sub> (Sandage 1954) is contributed by stars brighter than  $M_{pv} = -0.1$  mag., the level of the gap. For the remaining six clusters even more of the total light is attributable to stars in this magnitude range. In M<sub>2</sub>, for example, about 75 per cent seems to come from such stars.

In photographic wave lengths, of course, the number and distribution of stars in the blue horizontal sequence strongly affects the integrated magnitude. The integrated color index then depends on the nature of the horizontal sequence which was shown to vary considerably from clus-

ter to cluster. In addition, it is estimated that 80 per cent or more of the total luminosity of these clusters is due to stars in the magnitude range shown in the diagrams. Since the total brightness of the cluster is particularly sensitive to the brightest stars in the giant sequence, the integrated absolute magnitudes may not be an accurate gauge of the total number of stars in a cluster if the luminosity function differs at the bright end.

**4.4 Number of RR Lyrae stars and form of horizontal sequence.** Initial results on M<sub>3</sub> (Schwarz-schild 1940, Sandage 1953) and M<sub>92</sub> (Arp, Baum,

TABLE V. APPARENT DISTANCE MODULI

Cluster	RR Lyrae stars		Gap and color limits of gap		Adopted modulus and reddening		Estimated uncertainty
	$m_{pg}$	$m_{pv}$	$m_{pg}$	$m_{pv}$	$m_{pg}$	$m_{pv}$	
M <sub>3</sub>			15.72 .00 to	15.62 .18	15.72 no reddening	15.72	± .05
M <sub>5</sub>			15.24 .16 to	14.98 .36	15.24 .16 reddening	15.08	± .05
M <sub>13</sub>	14.83 (2 vars.)	14.57			14.83 .16 reddening	14.67	± .07
M <sub>10</sub>			15.22 .40 to	14.72 .60	15.22 .40 reddening	14.82	± .10
M <sub>15</sub>	15.90 (4 vars.)	15.79	15.89 .00 to	15.79 .20	15.90 no reddening	15.89	± .05
M <sub>2</sub>	16.17 (3 vars.)	16.05	16.21 .05 to	16.05 .27	16.19 .05 reddening	16.15	± .10
M <sub>92</sub>	15.30 (6 vars.)	15.12	15.20 .00 to	15.10 .20	15.20 no reddening	15.20	± .07

TABLE VI. MAGNITUDES OF BRIGHTEST STARS, INTEGRATED MAGNITUDES, AND FIELD STARS

	Brightest star (end of giant sequence)			Mean of 25 brightest stars			Integrated magnitude $M_{pg}$	Number of field stars expected brighter than $M_{pv}=0$ in each diagram		
	$M_{pg}$	Color index	$M_{pv}$	$M_{pg}$	Color index	$M_{pv}$				
M <sub>3</sub>	-1.4	+1.6	-3.0	-1.2	+1.2	-2.4	-8.5	10		
M <sub>5</sub>	-1.4	+1.4	-2.8	-1.2	+1.1	-2.3	-8.2	2.2		
M <sub>13</sub>	-1.3	+1.4	-2.8	-1.2	+1.1	-2.3	-8.1	2.1		
M <sub>10</sub>	-1.7	+1.4	-3.1	-1.2	+1.0	-2.2	-7.6	6.8		
M <sub>92</sub>	-1.7	+1.4	-3.1				-7.9	10.8		
M <sub>15</sub>	-2.0	+1.3	-3.3	-1.6	+1.1	-2.7	-8.6	1.6		
M <sub>2</sub>	-1.8	+1.2	-3.0	-1.4	+1.0	-2.4	-8.9	7.2		

and Sandage 1953) indicated that all the RR Lyrae variables fall in the horizontal sequence between C.I. = 0.0 and +0.2 mag. The RR Lyrae stars measured in M<sub>2</sub> and M<sub>15</sub> confirm this. The gap which was so defined has been drawn in the diagrams of the clusters. These show that the color width of 0.2 mag. is satisfactory and that non-variable stars do not occur in this region. The few non-variables which infringe over the edge of the gap as drawn are undoubtedly due either to probable errors of measurement or are small-amplitude variables (Walker 1955, Roberts and Sandage 1955).

As noted in the discussion of individual clusters, a large number of RR Lyrae cepheids in a given cluster is associated with a strong population on the red side of the gap. This is expressed quantitatively in Table IV. Here the number of stars in each diagram for an interval of 0.2 mag. in color index is given first for the blue side of the gap, then for the gap (0.2 mag. wide in color index and containing only RR Lyrae stars), and finally for the 0.2 mag. interval in color index to the red of the gap. Since the sequence is nearly horizontal in most clusters, this gives an approximation to the linear density function of stars

along the sequence. These sets of three numbers would suggest that a small number of stars on the red side of the gap signifies a sharply decreasing density function towards this end of the horizontal sequence. Presumably such a decreasing density function furnishes fewer candidates for variability in the gap.  $M_3$  is unusual in that the maximum star density occurs right in the RR Lyrae gap. It will be noted that it is not the total population of the branch but the population gradient across the gap which seems to correlate with the number of RR Lyrae stars; the total population of the branch can be quite large for clusters having few RR Lyrae stars (e.g.  $M_2$ ).

**4.5 Superposition of all seven diagrams.** Figure 13 shows the estimated line sequences from all the diagrams superposed. Where sequences coincided one line was used, so that seven lines are not present in all parts of the diagram.

There are about one hundred stars falling off the line sequences and plotted as individual points in Figure 13, whereas the line sequences represent about 2000 stars. Some of the stars falling off the sequences are undoubtedly field stars. In this connection, however, it can be stated that almost all of these stars that are bluer than C.I. = 0.0 mag. must be cluster members since field stars of this color are very rare. From Seares's (1925) counts the number of field stars to be expected in each diagram above the magnitude of the horizontal branch is given in Table VI. Altogether, about 41 field stars should be present above  $M_{pv} = 0$  in the composite diagram of Figure 13. Excluding the stars bluer than C.I. = 0, there are more than 50 stars falling off the sequences of Figure 13 in this magnitude range. Allowing for the occurrence of some field stars in the region of the sequences, an excess of 20 to 30 over the predicted 41 appears to be present. The type II population, as characterized by globular clusters, then contains some blue stars as bright as  $M_{pg} = -2$  mag. as well as possibly a small number of stars of intermediate color as bright as  $M_{pg} = -3$  mag.

The type II cepheids are indicated by circled crosses in Figure 13. They occupy a narrow region above the RR Lyrae cepheids. It is tempting to conclude that the high ratio of RR Lyrae cepheids to cepheids of period greater than one day (about 40 to 1 in these clusters) is due to the fact that the strong population of the horizontal branch places many more stars in the region of RR Lyrae variability than the scattering of blue stars above it places in the region of cepheid

variability with periods greater than one day. Only a few of the non-variable stars falling in the region of the type II cepheids have been checked for possible small-amplitude variation and none has been individually tested for cluster membership by means of radial velocities.

Of the variables near the bright end of the giant sequences, the three of reddest mean color index are long-period variables of 93, 103, and previously 106 days (it now seems irregular); the remaining members of this group appear to be irregular.

### 5.0 *The Giant Sequences.*

The absolute photographic and photovisual magnitude of the terminal point of the giant sequence in each cluster is listed in Table VI. This point is approximately equal to the brightest star in each cluster as can be seen from the diagrams. The adjoining columns in Table VI list the mean magnitude of the 25 brightest stars in each cluster, normalized to the same number of stars above  $M_{pv} = +0.6$  mag. as there are in the  $M_2$  diagram, 349. It is apparent that in these seven clusters there is a range of around half a magnitude in the absolute magnitude of both quantities. Distance criteria based on the brightest stars in such systems, however, would suffer more from the range in color index of these brightest stars, since the reddening would be uncertain by this amount and consequently the absorption by about four times this range.

In  $M_3$  and  $M_{92}$  the giant sequences were originally noted to be displaced parallel to each other in color index. The displacement of the giant sequences in the present clusters follows roughly the same pattern, except in  $M_{13}$ . In  $M_{13}$  the bright end of the giant sequence falls to the faint red side of the composite diagram (Figure 13) but then crosses over to the blue limit of the group of vertical branches near the gap. In  $M_2$  the upper end of the giant sequence falls in the brighter bluer group of giant sequences and then comes down a little to the red of the middle of the vertical branches near the gap.  $M_2$ , however, contains the sparsest calibration of the clusters and would be most subject to a small systematic error at fainter magnitudes. Therefore  $M_{13}$  is the only cluster which can be said with certainty to violate the parallel displacement in color index of the giant branches.

**5.1 Correlations with position of bright end of giant sequence.** The bright portion of the giant sequences in  $M_3$ ,  $M_5$ , and  $M_{13}$  average about

one-quarter magnitude redder at the same  $M_{pv}$  than in M<sub>10</sub>, M<sub>92</sub>, M<sub>15</sub>, and M<sub>2</sub>. The giant sequences in the first group tend toward an asymptotic limit at bright  $M_{pv}$  whereas those in the second group do not bend over much at the bright end. Nevertheless, the  $M_{pg}$  of the terminus gives another measure of the displacement of the giant branches, since all the giant branches are much more nearly horizontal when plotted with photographic magnitude as ordinate. The first column of Table VII shows that this criterion also tends to group the giant branches; the M<sub>3</sub>, M<sub>5</sub>, M<sub>13</sub> group having  $M_{pg} = -1.3$  to  $-1.4$  mag. and the remaining four ranging from  $-1.7$  to  $-2.0$  mag. Of course, it is not certain at this point whether this is an actual grouping or a chance sample from a gradation of giant-branch placements. For convenience of discussion, however, they will be considered here to be two discrete groups.

TABLE VII. CLUSTER CHARACTERISTICS WHICH ARE CORRELATED

Cluster	Giant branch $M_{pg}$	Mean period RR Lyr a, b	
M <sub>3</sub>	-1.4	.55 (124)	"Normal" Spectra (M <sub>3</sub> , M <sub>13</sub> ): cepheids in this group are more nearly like classical cepheids.
M <sub>5</sub>	-1.4	.54 (63)	
M <sub>13</sub>	-1.3	— (3)	
M <sub>4</sub>	—	.51 (17)	
M <sub>92</sub>	—	.55 (21)	
Mean		.545	
M <sub>92</sub>	-1.7	.63 (9)	"Abnormal" Spectra (M <sub>92</sub> , M <sub>15</sub> ): metal lines weak. W Virginis and RV Tauri cepheids in this group.
M <sub>10</sub>	-1.7	— (0)	
M <sub>2</sub>	-1.8	.63 (11)	
M <sub>15</sub>	-2.0	.65 (31)	
M <sub>53</sub>	—	.62 (17)	
$\omega$ Cen	—	.65 (77)	
M <sub>22</sub>	—	.63 (7)	
Mean		.643	

found in these clusters all occur in the second group (M<sub>10</sub>, M<sub>2</sub>, and  $\omega$  Cen). Finally, the three long-period variables of near-100-day period come from M<sub>3</sub>, M<sub>5</sub>, and M<sub>13</sub>. Members of the second group contain similar stars of both longer and shorter period, however, so that this correlation is an uncertain one.

The indicated correlations between the character of the variable stars, the placement of the giant sequences, and the nature of the spectra seem to be both reasonable and definite. The two distinct and separate groupings which are suggested, however, are puzzling. Even if study of additional globular clusters confirms this definite separation rather than a gradation between their physical properties, there seems to be no present hint as to its ultimate explanation.

Oosterhoff (1944) has shown that the mean period of all the RR Lyrae a and b cepheids, in clusters for which these data are available, has a value of either 0.51 to 0.55 or 0.62 to 0.65. The correlation between the mean period of the RR Lyrae a and b stars in a cluster and the placement of its giant branch is very good as seen in Table VII.

In the first group, Deutsch (unpublished results) has noted the spectra of stars in the giant sequences of M<sub>3</sub> and M<sub>13</sub> to be "normal" in the sense that they do not differ radically from high-velocity giants previously known. Two clusters in the second group, M<sub>92</sub> and M<sub>15</sub>, however, are noted to have abnormal spectra in that the metal lines are excessively weakened.

The three long-period cepheids (Arp, 1955) which have light-curves most resembling classical cepheids all occur in clusters of the first group (M<sub>3</sub> and M<sub>5</sub>). The six W Virginis type cepheids

#### 6.0 Remarks on the Zero Point of Absolute Magnitude.

At the inception of the present comparative study of globular clusters it was assumed that the RR Lyrae cepheids in all globular clusters have identical absolute magnitude and color indices. In retrospect, this assumption seems to have been partially justified, principally due to the following circumstances. When the zero point in each cluster is so chosen: 1) Characteristic features of all the color-magnitude diagrams above  $M_{pv} = +1$  occur at about the same absolute magnitude. 2) From different clusters those cepheids of period longer than one day having the same period and the same shape of light-curve, turn out to have the same absolute magnitude. Variation of the absolute magnitudes of

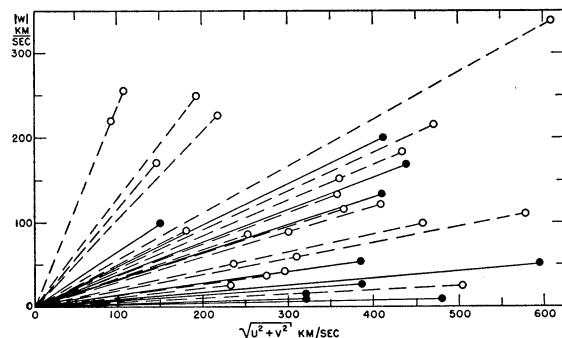


Figure 19. Velocities of 33 high-velocity dwarfs in and perpendicular to the plane of the galaxy. Filled circles represent normal dwarfs; open circles, subdwarfs.

the RR Lyrae stars from cluster to cluster of more than  $\pm 0.1$  mag. would have vitiated the multiple period-luminosity observed (Arp 1955). 3) Long-period variables, around 100 days, turn out to have closely the same magnitudes (unpublished).

This appears to fix all the RR Lyrae stars at the same luminosity to within  $\pm 0.1$  mag., i.e., the uncertainty involved in estimating individual zero points.

It was also hoped that an absolute calibration of the RR Lyrae stars could be obtained by superposing the main sequence in globular clusters upon the main sequence in the neighborhood of the sun, since stars as faint as  $M_{pv} = +5$  or thereabouts presumably have not had time in which to evolve an appreciable distance in the color-magnitude plane. This assumed that the main sequence in the globular clusters provides a zero point as good as the RR Lyrae stars. Although the above procedure gave a reasonable value for the RR Lyrae stars in M<sub>3</sub> (Sandage 1953), the main sequence in M<sub>92</sub> (Arp, Baum and Sandage, 1953) fell somewhat below that in M<sub>3</sub> and recent measures by Baum (1954) show the main sequence in M<sub>13</sub> to be more than a magnitude below even that position.

Observationally, however, it is clear that this situation is to be expected. Fricke (1949) lists 33 high-velocity dwarfs, of which 10 turn out to be main-sequence dwarfs and 23 subdwarfs. Among these possible type II objects the ones which inhabit the halo regions, above the plane of the galaxy, may be selected on the basis of

their velocity component perpendicular to the plane, W. Figure 19 is constructed from Fricke's data and shows how the ratio of subdwarfs to dwarfs increases as we go to higher velocities perpendicular to the plane.

Clearly it is the subdwarfs that are the stars more characteristic of the halo region, much nearer the region of the globular clusters, since their observed velocities indicate that they are the stars which are passing through the solar neighborhood from those regions. The position of the globular-cluster main sequence then does not appear to be necessarily that of the main sequence in the vicinity of the sun. A fit cannot be made unless there is a unique subdwarf sequence (indicated not to be the case from M<sub>3</sub>, M<sub>92</sub>, and M<sub>13</sub>). The method seems only applicable in the event that specific nearby stars can be equated to the main sequence in a particular globular cluster.

The differences in the positions of the main sequences of the globular clusters in the color-magnitude diagram might be connected with initial chemical composition if age cannot be a factor at such low luminosity. Such chemical composition differences could conceivably enter into the differences previously mentioned as appearing in the spectra of the brighter globular-cluster stars.

#### REFERENCES

- Arp, H. C. 1955, *A. J.* **60**, 1.
- Arp, H. C., Baum, W. A. and Sandage, A. R. 1952, *A. J.* **57**, 4.
- . 1953, *A. J.* **58**, 4.
- Baade, W. 1944, *Ap. J.* **100**, 137.
- Baum, W. A. 1954, *A. J.* **59**, 422.
- Christie, W. H. 1940, *Ap. J.* **91**, 8.
- Cuffey, J. B. 1943, *Ap. J.* **98**, 49.
- Fricke, W. 1949, *A. N.* **278**, 51.
- Greenstein, J. L. 1939, *Ap. J.* **90**, 387.
- Hachenberg, O. 1939, *Zs. Astroph.* **18**, 49.
- Oosterhoff, P. T. 1944, *B. A. N.* **10**, 55.
- Roberts, M. S. and Sandage, A. R. 1955, *A. J.* **60**, 185.
- Sandage, A. R. 1953, *A. J.* **58**, 61.
- . 1954, *A. J.* **59**, 162.
- Schwarzschild, M. 1940, *Circ. Astr. Obs. Harvard No.* 437.
- Seares, F. H. 1925, *Ap. J.* **62**, 320.
- Shapley, H. 1917, *Ap. J.* **45**, 123.
- Walker, M. F. 1955, *A. J.* **60**, 197.

*Mount Wilson and Palomar Observatories,  
Carnegie Institution of Washington,  
California Institute of Technology,  
Pasadena, Calif.  
1954 December.*